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Optimal Placement of Voltage Regulators for Photovoltaic Hosting Capacity Maximization

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Abstract**-- With the rapidly growing integration of distributed photovoltaic (PV) generation, the need to maximize the hosting capacity (HC) of PV generation in distribution networks (DN) has become a major concern. To maximize the size of PV power absorbed by the system while maintaining required levels of power quality and system stability, this paper has proposed a deterministic optimization based optimal voltage regulator (VR) placement model. This model maximizes the installed capacity of PV generation and minimizes the investment costs of VR, while maintaining the bus voltage within their operating limits. Mathematically, this optimal VR placement problem is formulated as a nonlinear program, then the linearization methods are employed to transform the model to a mix-integer linear program. Finally, the effectiveness of the proposed method is demonstrated using a modified IEEE 33-bus distribution system.**

Index Terms**-- PV generation, hosting capacity, distribution networks, installed capacity, VR placement, linearization method, mix-integer linear program (MILP).**

Nomenclature

A. Sets and Indices

i / N	Index/set of distribution nodes.
k	Index of photovoltaic generation units.
$\gamma(i)$	Set of child nodes of the node i .
t / T	Index/set of time slots.

B. Variables

P_{it} / Q_{it}	Active/Reactive power flow through the branch between node $i-1$ and node i .
q_{it}^{SVC}	Compensation rate of static VAR compensator of node i at t .
a_i^{VR}	Binary decision variable flagging VR placement of node i .
V_{it}	Bus voltage of node i at t .
V_{mt}	Bus voltage of the position m between $i-1$ and i where VR is installed at t .
$\underline{s}_{it}$ / $\overline{s}_{it}$	Slack variables for voltage limitation.
E_k^{PV}	Installed PV generation capacity of unit k .

C. Parameters

w^{PV}	Weight factor of Installed PV generation capacity.
w^{VR}	Weight factor of VR investment cost.
$w^{Penalty}$	Weight factor of penalty cost of slack variables for voltage magnitude.
$C^{Penalty}$	Penalty cost for voltage violations.
C_{inv}^{VR}	Objective function coefficient associated to the VR investment cost.
M_{inv}^{VR}	Maximum allowed total VR investment cost.
λ_t^{PV}	PV power output level (ratio of installed PV generation capacity) at time t .
r_i / x_i	Resistance/Reactance of the branch between node $i-1$ and node i .
p_{it}^d / q_{it}^d	Active/Reactive net load at node i at t .
$\underline{V}_i$ / $\overline{V}_i$	Lower/Upper bound of voltage at node i .
$\underline{E}_k^{PV}$ / $\overline{E}_k^{PV}$	Lower/Upper bound of PV generation of unit k .
$\underline{Q}_i^{SVC}$ / $\overline{Q}_i^{SVC}$	Lower/Upper bound of VR compensation level.

I. Introduction

THE increased global-installed capacity of renewable distributed generation (RDG), especially PV generation, is encouraged worldwide as a promising solution to meet energy constraints and environmental constraints. The widespread use of PV generation technologies can benefit the modern power system such as energy cost savings, power loss reduction and system reliability improvement [1]. However, PV generation installations may disrupt normal operating conditions of the power system, like thermal limits and voltage profile limits, since the distribution systems are designed for centralized power generation with optimization for unidirectional power flow [2]. Besides, due to the intermittency of renewable energy outputs, which limit the ability of the distribution network to connect PV generators.

PV hosting capacity is defined as the maximum total PV capacity that a distribution network can accommodate without violating operational constraints, especially bus voltage constraints. PV hosting capacity depends on kinds of factors, like PV type, distribution network (DN) characteristics,

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limiting criteria defined by the DN operator [3]. And PV penetration is limited by some reasons, such as bus voltage violations, line current limits, control schemes, and impacts on protection systems of DN [4, 5]. To efficiently integrate PV in DN, several methods have been proposed. Reference [6] evaluates the ability of a distribution system to accommodate DGs and proposes an maximum hosting capacity (MHC) method by modifying operation parameters of existing components. In [7], the potential of using battery energy storage systems in the public low-voltage distribution system is investigated to postpone upgrades to increase the PV penetration level. In [8], a significant increase in allowable integration of DG can be achieved by controlling reactive power compensation level of the DG unit itself. Reference [9] explores how the DG hosting capacity can be improved by means of static and dynamic network reconfiguration and multi-period optimal power flow (MP-OPF)-based method is presented to maximize the DG hosting capacity.

The optimal placement of VR in the DN is an interesting alternative for solving the aforementioned technical problems. The VR is mainly installed for voltage control, which is widely used in traditional distribution system and has an influence on MHC. Therefore, in this work, VR placement is selected to represent voltage regulation. The decreasing VR investment cost together with longer service life renders this solution more and more attractive for the electricity utilities.

Therefore, in this paper, an optimal VR planning model is proposed for maximizing PV generation hosting capacity in distribution system while maintaining the bus voltage within their operating limits. This placement model is formulated as a deterministic program problem, where optimal VR placement decisions can be acquired by maximizing the installed PV generation capacity as well as minimizing the VR investment cost and voltage deviation, subject to simplified Distflow equations. A general linearization technique is employed to transform the original nonlinear program into a mixed integer linear program (MILP). The deterministic operation scenario of PV outputs and load demand are developed from the historical data. Finally, the proposed model is tested on the modified 33-node distribution system to verify its effectiveness.

The rest of this paper is organized as follows. Section II gives the mathematical formulation of the deterministic optimization based optimal VR planning model. Section III describes the case study to evaluate the effectiveness of the proposed approach and then followed by the detailed analyses and discussion of results. Finally, concluding remarks are included in Section IV.

II. Mathematical Formulation

In this section, we introduce the widely used linearized power flow equations of distribution network and provide the deterministic formulation of VR placement problem. Then, the mathematical formulation is transformed to be a mixed integer linear program to maximize hosting capacity of PV generation.

A. Distribution Network Model

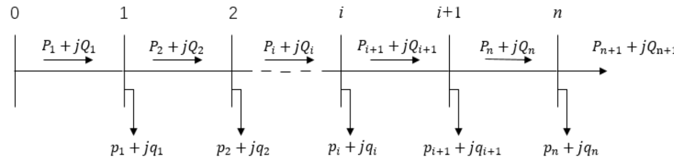

Fig. 1. Diagram of a radial distribution system.

Considering a distribution system as shown in Fig. 1, there are n buses that are indexed by $i = 0,1,2,...,n$. In order to deal with the problem of nonlinearity, the linear version of the DistFlow equations is proposed and justified in [10] and [11]. Specifically, the complex power flows at 14each node i can be described using simplified DistFlow equations as follows,

$$P_{i+1}=P_i - p_i, \forall i \in N \tag{1a}$$

$$Q_{i+1}=Q_i - q_i, \forall i \in N \tag{1b}$$

$$V_{i+1}=V_i - \frac{r_{i+1}P_{i+1} + x_{i+1}Q_{i+1}}{V_0}, \forall i \in N \tag{1c}$$

$$p_i = p_i^d - p_i^g, \forall i \in N \tag{1d}$$

$$q_i = q_i^d - q_i^g, \forall i \in N \tag{1e}$$

where equation (1a) describes the active power flow, equation (1b) describes the reactive power flow, and equation (1c) describes the voltage transmit along the branch.

B. Linearization of VR Model

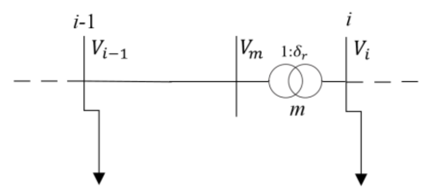

Fig. 2. One line diagram of a distribution feeder with a VR.

Fig.2 shows one line with a VR installed in the radial distribution feeder. When a VR device is installed at the point m , this distribution branch can be divided into two parts, one part is from node $i-1$ to point m and another part is from point m to node i . Then the voltage transmit along the branch can be described as,

$$V_{mt}=V_{i-1t} - \frac{r_i P_{it} + x_i Q_{it}}{V_0}, \forall i \in N, \forall t \in T \tag{2a}$$

$$\delta_r V_{mt}=V_{it}, \forall i \in N, \forall t \in T \tag{2b}$$

$$\delta_r = 1 + \Delta_{tap} V_{step} a_i^{VR}, \forall i \in N \tag{2c}$$

$$\Delta_{tap} \in [-16,16], \text{integer} \tag{2d}$$

$$V_{step} = 0.00625\,\text{p.u.} \tag{2e}$$

$$a_i^{VR} \in [0,1], \text{binary}, \forall i \in N \tag{2f}$$

where V_{mt} represents the voltage magnitude of the point m at time t and δ_r represents the turns ratio of the VR in this branch. Δ_{tap} and V_{step} denote the tap-position change and the step voltage per tap change of VR respectively, and a_i^{VR} denotes whether a VR is placed ($a_i^{VR} = 1$) on the branch i or not ($a_i^{VR} = 0$). Then the equation (2b) can be written as,

$$V_{mt}+0.00625\Delta_{tap}a_i^{VR}V_{mt}=V_{it},\forall i\in N,\forall t\in T \tag{2g}$$

It can be noted that this nonlinear VR placement problem is nonconvex and cannot be solved by commercial solvers as the trilinear term $\Delta_{tap}a_i^{VR}V_{mt}$ exists in the above equation (2g). To linearize this trilinear term, we follow two steps. In the first step, we make an assumption that the integer variable tap-position change Δ_{tap} can be regarded as a continuous variable. This is because the step voltage per tap change V_{step} is quite small, the mismatch of voltage magnitude calculation can be neglected. So the equation (2g) can be written as two inequations considering boundaries of tap-position change Δ_{tap} as follows,

$$V_{mt}-0.1a_i^{VR}V_{mt}\le V_{it},\forall i\in N,\forall t\in T \tag{2h}$$

$$V_{it}\le V_{mt}+0.1a_i^{VR}V_{mt},\forall i\in N,\forall t\in T \tag{2i}$$

There is still bilinear term $a_i^{VR}V_{mt}$ in inequations (2h)-(2i), which can be solved efficiently. Thus, in the second step, we introduce an auxiliary continuous variable z_{it}^{VR} to denote $z_{it}^{VR}=a_i^{VR}V_{mt}$, which contains a binary variable a_i^{VR} and a continuous variable V_{mt}. To replace $a_i^{VR}V_{mt}$ by z_{it}^{VR}, the following four inequations should be added,

$$-a_{it}^{VR}\overline{V}_i+z_{it}^{VR}\le 0,\forall i\in N,\forall t\in T \tag{2j}$$

$$a_{it}^{VR}\underline{V}_i-z_{it}^{VR}\le 0,\forall i\in N,\forall t\in T \tag{2k}$$

$$-a_{it}^{VR}\underline{V}_i+z_{it}^{VR}\le V_{mt}-\underline{V}_i,\forall i\in N,\forall t\in T \tag{2l}$$

$$-a_{it}^{VR}\overline{V}_i+z_{it}^{VR}\le -V_{mt}+\overline{V}_i,\forall i\in N,\forall t\in T \tag{2m}$$

Finally, equation (2b) can be formulated as two inequations as follows,

$$V_{mt}-0.1z_{it}^{VR}\le V_{it},\forall i\in N,\forall t\in T \tag{2n}$$

$$V_{it}\le V_{mt}+0.1z_{it}^{VR},\forall i\in N,\forall t\in T \tag{2o}$$

C. Mathematical Formulation of VR Placement Problem

In this subsection, a deterministic optimization based framework is proposed. To maximize the installed PV generation capacity that can be accommodated by the distribution network in given locations and to keep the DN voltage deviation within the requirement, the optimal placement of VR is proposed in this work. The detailed planning model can be described as follows,

1) Objective function

The focus of this paper is on maximum the hosting capacity of distribution system through optimal VR placement. In this paper, an optimization problem is formulated which maximizes the installed capacity of PV generation by objective function (3a) and minimizes the VR investment cost as well as penalty cost of slack variables for voltage magnitude by objective function (3b) in the distribution system while meeting the voltage constraints, defined as follows,

$$\text{Max } w^{PV}\sum_{k\in\gamma(i)}E_k^{PV} \tag{3a}$$

$$\text{Min } w^{Penalty}\sum_i C^{Penalty}\sum_t(\underline{s}_{it}+\overline{s}_{it})+w^{VR}\sum_i C_{inv}^{VR}a_i^{VR} \tag{3b}$$

The maximization problem in (3a) is changed to a minimization the then combined with (3b) to form the new objective function (3c) as follows,

$$\text{Min}-w^{PV}\sum_{k\in\gamma(i)}E_k^{PV}+w^{Penalty}\sum_i C^{Penalty}\sum_t(\underline{s}_{it}+\overline{s}_{it})+w^{VR}\sum_i C_{inv}^{VR}a_i^{VR} \tag{3c}$$

2) Constraints

$$\text{s.t. (2a), (2j)-(2o)} \tag{4a}$$

$$P_{i+1t}=P_{it}-p_{it}^d+p_{kt}^{PV},\forall i\in N,\forall k\in\gamma(i),\forall t\in T \tag{4b}$$

$$Q_{i+1t}=Q_{it}+q_{it}^{SVC}-q_{it}^d,\forall i\in N,\forall t\in T \tag{4c}$$

$$p_{kt}^{PV}=\lambda_t^{PV}E_k^{PV},\forall k\in\gamma(i),\forall t\in T \tag{4d}$$

$$\underline{V}_i-\underline{s}_{it}\le V_{it}\le\overline{V}_i+\overline{s}_{it},\forall i\in N,\forall t\in T \tag{4e}$$

$$\underline{s}_{it}\ge 0,\overline{s}_{it}\ge 0,\forall i\in N,\forall t\in T \tag{4f}$$

$$\underline{E}_k^{PV}\le E_k^{PV}\le\overline{E}_k^{PV},\forall k\in\gamma(i) \tag{4g}$$

$$\underline{Q}_i^{SVC}\le q_{it}^{SVC}\le\overline{Q}_i^{SVC},\forall i\in N,\forall t\in T \tag{4h}$$

$$\sum_i C_{inv}^{VR}a_i^{VR}\le M_{inv}^{VR},\forall i\in N \tag{4i}$$

where constraint (4a) is discussed in the previous subsection. Constraints (4b) and (4c) are linearized DistFlow equations. λ_t^{PV} appearing in (4d) denotes the rate of installed capacity of PV generation. Non-negative slack variables $\underline{s}_{it}$ and $\overline{s}_{it}$ are included in (4f) to relax bus voltage constraints (4e). Constraints (4g) and (4h) describe the installed PV generation capacity limitation and SVC operation limitation respectively. Constraint (4i) describes that the total VR investment cost cannot exceed a predefined limit from practical view.

III. CASE STUDY

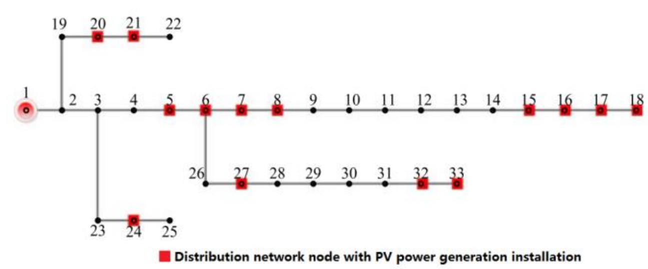

Fig. 3. Modified IEEE 33-bus test distribution system.

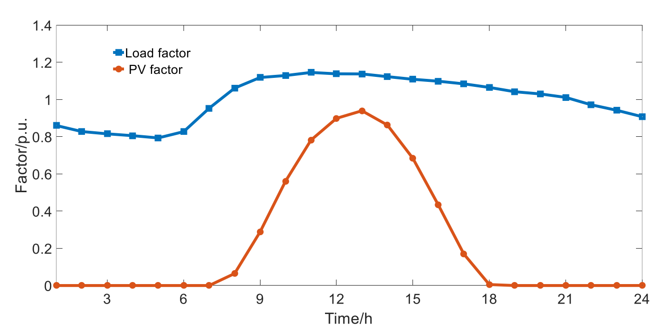

Fig. 4. Load factor and the PV output curve.

In this section, the proposed methodology is implemented on the modified IEEE 33-node distribution system [12] to seek the optimal investment locations of VRs. Fig. 3 shows the test distribution system, which is modified by installing 14 PV distributed generators with the maximum capacity of each PV base being 300 kW, located at node 5, 6, 7, 8, 15, 16, 17, 18, 20, 21, 24, 27, 32 and 33 respectively. The operation cost of wind power generation is assumed to be zero in this paper. Besides, there are five static VAR compensators (SVCs) in this test system, located at node 5, 6, 15, 20 and 32 respectively. Fig.4 shows the 24-hour profiles of the load factor and PV output curve in this test modified DN. The proposed deterministic optimization model is formulated as a mixed integer linear program (MILP), which is solved with CVX [13].

A. Optimization Results

TABLE I
RESULTS OF MAXIMUM PV HOSTING CAPACITY

Location (Node)	PV Size (p.u.)
5	0.27
6	0.01
7	0.01
8	0.01
15	0.07
16	0.01
17	0.01
18	0.01
20	0.17
21	0.01
24	0.10
27	0.33
32	0.17
33	0.01

TABLE II
RESULTS OF OPTIMAL VR LOCATIONS

Location (Node)	2, 3, 4, 5, 6, 9, 10, 19, 23, 26, 28

Table I shows the results of maximum allocation of PV hosting capacity in the test distribution grid. It can be see from this table that the MHC of PV generation in this test distribution network is 1.186 p.u.. Table II shows the results of optimal VR placement. Under the condition of high PV power penetration, voltage regulation can be achieved by adjusting the tap position of the installed VRs.

TABLE III
COMPARISON OF PV GENERATION HOSTING CAPACITY WITH VRS AND WITHOUT VRS

	With VR	Without VR
PV Hosting Capacity (p.u.)	1.186	0.88

It is observed from Table III that the improvement of PV generation MHC can be achieved after optimal VR devices placement.

B. Investigation of Voltage Profiles

Fig. 5 demonstrates the detailed information of voltage profiles in the test DN. The mean value of voltage profiles of 24 hours of all nodes is 0.9928 p.u. and the maximum voltage deviation is 0.05 p.u.. This means all voltage profiles are within the standard. Thus, with optimal placement of VR, PV hosting capacity is maximized while satisfying steady state voltage limits.

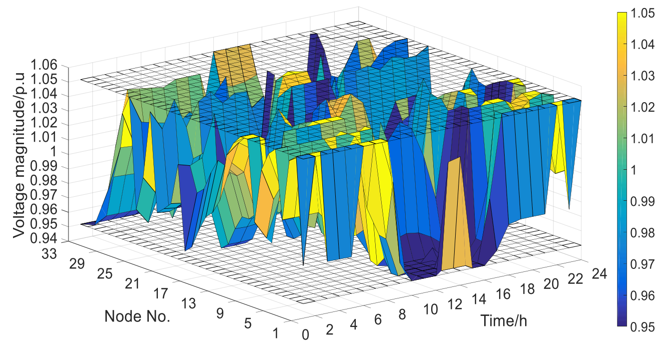

Fig. 5. Voltage magnitude.

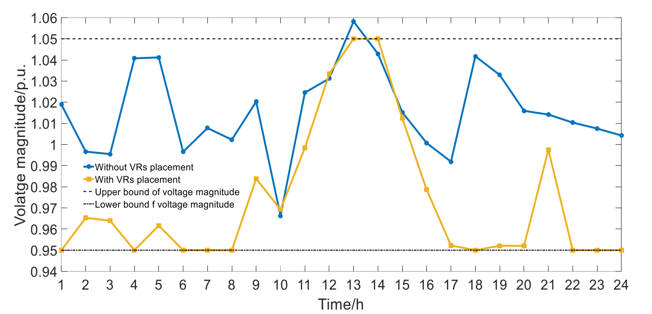

Fig. 6. Voltage profiles comparison with maximum allowable PV power penetration.

Fig. 6 gives the voltage profiles of the test distribution system under the condition of maximum PV generation capacity installation. There are two profiles shown in this figure: 1) voltage profile on node 23 of each time slot with VRs placement (yellow curve); and 2) voltage profile on node 23 of each time slot without VRs placement (blue curve). Under the condition of maximum PV power penetration, the voltage deviations become large in this test distribution system. After optimal VRs installation, as the voltage profile 1) shown, the bus voltage magnitudes of 24 hours are within the standards and the largest voltage deviation ($(V_{it}-1)^2$) of this voltage profile is less than 0.0025 p.u.. However, without VRs installation, shown as the profile 2), voltage magnitude of this node exceeds the upper limits at 13pm. Besides, the daily voltage magnitudes of profile 2) becomes more fluctuating that of profile 1) and the maximum voltage deviation of this voltage profile 2) is 0.0034 p.u., which is also larger than that of the profile 1). The reason is that PV output is high during the daytime, and without VR to adjust voltage magnitude accordingly, the voltage magnitudes may exceed the basic limitations and voltage deviations become quite large in some time slots.

C. Sensitivity Analysis

In this section, a sensitivity analysis of the VR installation number is conducted to explore how MHC of PV generation is influenced by the number of VR devices in the DN. Fig. 7 demonstrates the impact of installed VR devices number on the maximum PV generation hosting capacity in the test distribution grid. It can be observed from this figure that the maximum PV generation hosting capacity can be improved with the increasing installed number of VR devices. Therefore, changing the installed VR number helps to achieve PV hosting capacity maximization significantly at first but the enhancement of MHC levels off when installing enough well-located VRs in the test system. Note that there is still some room for hosting capacity enhancement even without VRs placement. Thus, larger number of well-located VR devices cannot help to improve efficiently MHC of PV generation but can lead to high VRs investment cost.

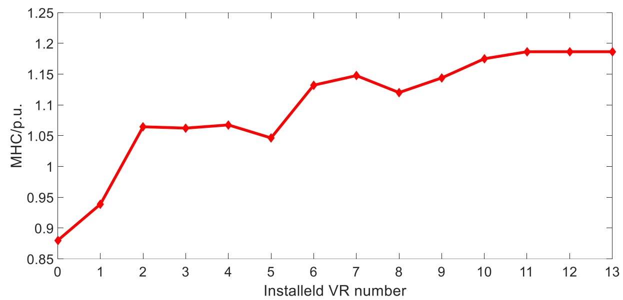

Fig. 7. Impact of installed VR number on PV hosting capacity enhancement.

IV. Conclusion

This paper presents a VRs installation method, then a MILP-based solution approach is adopted to obtain the optimal locations of VR devices for PV generation hosting capacity maximization while maintaining steady state voltage limitations. The objective function is to maximize total installed distributed PV generation capacity and minimize the investment cost of VR devices, a proper penalty function is used to consider the limiting factor—voltage constraints. Results on a modified IEEE 33-bus distribution system verify the effectiveness of the proposed deterministic optimization based method by revealing that maximum PV hosting capacity can be achieved by optimal VRs placement under voltage limits. To explore the impact of installed VRs number on MHC, sensitive analysis is performed, which offers a practical method to optimally determine VRs placement. The proposed framework can also be employed to account for other RDG hosting capacity limiting factors, such as voltage deviation constraints and thermal constraints. Besides, the results need further investigation by more operation scenarios.

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