Enhancing Photovoltaic Hosting Capacity—A Stochastic Approach to Optimal Planning of SVC in Distribution Network

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***Abstract*—To improve photovoltaic (PV) hosting capacity of distribution networks (DN), this paper proposes a novel optimal static VAR compensator (SVC) planning model which is formulated as a two-stage stochastic programming problem. Specifically, the first stage of our model determines the SVC planning decisions and the corresponding PV hosting capacity. In the second stage, the feasibility of the first stage results is evaluated under different uncertainty scenarios of load demand and PV output to ensure no constraint violations, especially no voltage violations. In addition, we simultaneously consider the minimization of SVC planning cost and the maximization of PV hosting capacity by formulating a multi-objective function. To improve the computational efficiency, a solution method based on Benders decomposition is developed by decomposing the two-stage problem into a master problem and multiple subproblems. Finally, the effectiveness of the proposed model and solution method is validated on modified IEEE 37-node and 123-node distribution systems.**

***Keywords*—Photovoltaic (PV) hosting capacity, distribution networks (DNs), static VAR compensator (SVC) planning model, two-stage stochastic programming problem, uncertainty scenarios of load demand and PV output, Benders decomposition**

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Nomenclature

Sets and Indices

i / N	Index/set of distribution nodes.
m	Index of nodes with PV generation installation.
$N^{PV}(i)$	Set of child nodes of the node i with PV generation units.
t / T	Index/set of time periods.
s / S	Index/set of scenarios.

Variables

P_{its} / Q_{its}	Active/Reactive power flow through the branch between node $i-1$ and node i in period t and scenario s.
V_{its}	Node voltage at node i in period t and scenario s.
a_i^{SVC}	Binary decision variable flagging SVC installation at node i or not.
Q_i^{SVC}	SVC installation capacity at node i.
q_{its}^{SVC}	Reactive power support of SVC at node i in period t and scenario s.
E_m^{PV}	PV hosting capacity allocated to node m.
$\overline{s}_{its}$ / $\underline{s}_{its}$	Slack variable for upper/lower bound of voltage magnitude at node i in period t and scenario s.

Parameters

w^{PV}	Weighting factor of PV hosting capacity.
w^{SVC}	Weighting factor of the SVC planning cost, including SVC investment cost and SVC operation cost.
C_F^{SVC}	Objective function coefficient associated to the fixed investment cost of SVC ($).
C_V^{SVC}	Objective function coefficient associated to the varying operation cost of SVC ($/h).
$C^{Penalty}$	Objective function coefficient associated to the penalty cost for voltage violation ($/p.u.).
p_s	Probability of scenario s occurrence.
N_{inv}^{SVC}	Maximum allowed total SVC installation number.
ξ_{ts}^{PV}	PV output factor (ratio of PV hosting capacity) in period t and scenario s, $\xi_{ts}^{PV} \in [0,1]$
p_{mts}^{PV}	PV output of unit m in period t and scenario s.

1. Introduction

The proliferation of renewable distributed generation (RDG), especially photovoltaic (PV) generation, is a promising strategy to address the worldwide energy and environmental concerns. The widespread use of PV generation technologies has a lot of benefits such as reducing energy cost and emission, deferring upgrade of transmission network, and relieving reliance on fossil fuels [1, 2]. On the other hand, the overuse of PV generation may disrupt normal operating conditions of the power system, like overload of distribution lines and voltage constraints, due to the lack of advanced control schemes [3, 4]. To maintain the reliable and secure operation of power systems, large amount of PV curtailment has been observed across the world [5], particularly in China [6]. Therefore, it is critically import to improve the PV hosting capacity of power systems, especially the distribution networks (DNs).

PV hosting capacity is defined as the maximum total PV capacity that a DN can accommodate without violating operational constraints, especially node voltage constraints. Various factors could impact PV hosting capacity like PV type, DN characteristics, and limiting criteria defined by the DN operators [7-9]. Consequently, it is challenging to assess the PV hosting capacity of a DN. The simulation based approach is mostly used to evaluate the PV hosting capacity [10-13]. For example, Monte Carlo simulation based stochastic analysis is employed to estimate PV hosting capacity in [13]. There are also some works focusing on the improvement of PV hosting capacity. Ref. [14] investigates the potential of battery energy storage systems to improve the PV penetration level. Ref. [15] develops a reactive power control method using RDG units to enhance the integration of renewable energy. Ref. [16] explores how the RDG hosting capacity can be improved by means of static and dynamic network reconfiguration. Ref. [17] uses active-management strategies (AMSs) to improve the RDG hosting capacity. However, most works focus on enhancing the PV hosting capacity based on the short-term operation strategies and overlook the impact of the long-term planning, which results in a very limited enhancing capability. Therefore, we endeavor to improve the PV hosting capacity from the perspective of long-term planning.

The installation of SVC in the DN is envisioned to be an effective means to enhance the PV hosting capacity, since SVC is capable of voltage regulation by absorbing or releasing reactive power. Traditionally, capacity bank (CB) is utilized to compensate reactive power in DNs due to its relatively low installation cost and maintenance cost. However, CB can only release reactive power with discontinuous adjustment. Besides, over-frequent action of CB will lead to the reduced lifetime. By contrast, SVC is capable of consuming and compensating reactive power continuously with fast reaction in response to the voltage variations. Thus, SVC can be employed to alleviate the overvoltage violations caused by the high PV generation. Hence, the placement of SVC has a considerable influence on PV hosting capacity. However, classical SVC planning studies [18-21] overlook the potential of SVC planning for PV hosing capacity enhancement. For example, the SVC planning problem in [18] only focuses on addressing the challenge of increasing load demand by strengthening the voltage regulation capability. Instead of improving the voltage regulation performance, our work mainly focuses on maximizing the PV hosting capacity of the DN with optimal planning of SVC.

There are various uncertainties in the DN, e.g. uncertain load demand and renewable energy output. Robust optimization (RO) [22] and stochastic programming [23] are two typical methods to tackle the uncertainties. Compared with the stochastic programming solutions, the solutions of RO are often considered to be over-conservative since RO gives too much emphasis on the worst-case scenario whose occurrence probability is relatively low. Generally, stochastic programming is adopted to model the power system planning problem by minimizing the expected cost over the multiple representative uncertainty scenarios subject to all practical constraints. Thus, stochastic programming is more robust than the deterministic optimization but less conservative than RO. We hence adopt stochastic programming to formulate our planning problem.

In this paper, we propose a novel optimal SVC planning model based on stochastic programming aiming at maximizing the PV hosting capacity of the DN. In particular, the model is formulated as a two-stage problem, where the first stage is to determine the PV hosting capacity and the SVC planning decisions, and the second stage is to ensure there is no operation constraints violation for any considered uncertainty scenarios given the predetermined first stage results. In addition, we develop an efficient solution method based on the Benders decomposition to solve this two-stage stochastic problem. The effectiveness of the proposed model and the solution method is verified on the modified 33-node and 123-node distribution systems. The major contributions are summarized in threefold as below,

1) This paper proposes an effective and efficient way to enhance PV hosting capacity, which plays an important role in identifying the capability of a DN to accommodate PV generations. Considering that SVC is widely used in power system, this work investigates the potential benefits of optimal SVC planning for improving PV hosting capacity by offsetting the voltage rise problems caused by PV integrations.

2) PV hosting capacity is difficult to be evaluated. Empirically, it is assessed using Monte Carlo simulation based approaches like [13]. However, simulation-based approaches are time-consuming for the large systems and hardly applicable in studying PV hosting capacity enhancement. In contrast, we originally model the PV hosting capacity as a decision variable in the optimization context. Specially, we propose a novel two-stage SVC planning problem based on the stochastic programming and incorporate the PV hosting capacity into the objective function. Thus, we can achieve a tradeoff between PV hosting capacity maximization and the SVC planning cost minimization.

3) This paper develops a Benders decomposition based solution method to efficiently solve the proposed two-stage planning problem. To our best of acknowledge, this is the first study to employ Benders decomposition algorithm to solve the two-stage SVC planning problem for PV hosting capacity improvement by far.

The rest of this paper is organized as follows. Section 2 gives the mathematical formulation of the stochastic programming based optimal SVC planning model. Section 3 describes the solution methodology based on Benders decomposition. Section 4 describes the case studies to evaluate the effectiveness of the proposed planning model and solution approach and then followed by the detailed analyses and discussion of results. Finally, concluding remarks are included in Section 5.

2. Problem Formulation

2.1. Two-stage Stochastic Framework

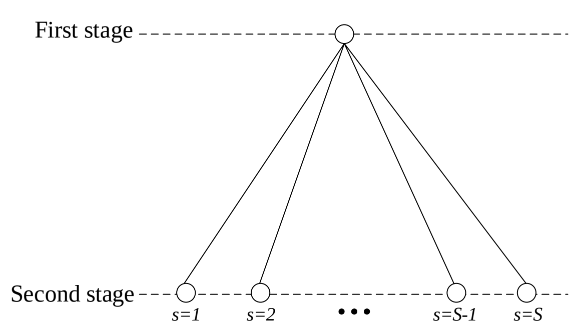


Fig. 1 Two-stage stochastic decision framework of the proposed SVC planning problem.

Fig. 1 depicts the two-stage stochastic framework of the proposed SVC planning problem in this paper. Practically, the first stage decision variables are determined before the actual realization of the uncertain load demand and PV output, including the sitting and sizing of SVC as well as evaluation of PV hosting capacity. On the other hand, the second stage decision variables represent the operation decisions and thus depend on the uncertain realization. Therefore, we model the SVC planning problem as a two-stage stochastic programming problem, where the first-stage variables are named as *here-and-now* decisions and the second-stage variables are called *wait-and-see* decisions.

There are various uncertainties in the DN, e.g. uncertain load demand and renewable energy output. In this work, uncertainties of PV output and load demand are taken into consideration. These uncertainties are represented as the form of scenarios based on the historical data obtained from [24]. Specifically, we use about 3500 daily scenarios of PV output and load demand in Nordic countries over the past ten years, respectively. The original numerous scenarios need to be reduced to representative scenarios as the inputs of the stochastic planning process. The derived numerical scenarios should be reduced to a set of representatives to facilitate the stochastic programming. Here, a backward-reduction algorithm based on Kantorovich Distance (KD) [25] is employed due to its capability of generating the associated weights (probabilities) of the selected scenarios, which can distinguish the significance of the inputs of the subsequent stochastic planning stage. The procedure of this scenario reduction method is interpreted in [25].

2.2. DistFlow Model

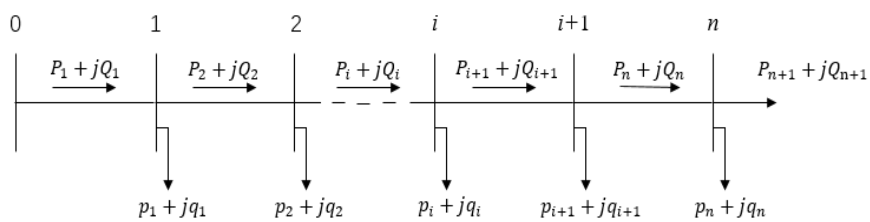


Fig. 2 Diagram of a radial distribution system.

Consider a distribution system with $n+1$ nodes indexed by $i = 0,1,2,...,n$ as shown in Fig. 2. The power flow equations can be described using DistFlow model [26, 27] as follows,

$$P_{i+1} = P_i - r_i \frac{P_i^2 + Q_i^2}{V_i^2} - p_i, \forall i \in N \tag{1a}$$

$$Q_{i+1} = Q_i - x_i \frac{P_i^2 + Q_i^2}{V_i^2} - q_i, \forall i \in N \tag{1b}$$

$$V_{i+1}^2 = V_i^2 - 2(r_{i+1}P_{i+1} + x_{i+1}Q_{i+1}) + (r_{i+1}^2 + x_{i+1}^2)\frac{P_{i+1}^2 + Q_{i+1}^2}{V_i^2}, \forall i \in N \tag{1c}$$

$$p_i = p_i^d - p_i^g, \forall i \in N \tag{1d}$$

$$q_i = q_i^d - q_i^g, \forall i \in N \tag{1e}$$

where equations (1a) and (1b) describe the active and reactive power balance at each node, respectively; Equation (1c) describes the voltage relationship between two adjacent nodes. In order to reduce the complexity, the linearized DistFlow equations are proposed by neglecting the high-order terms in (1a)-(1c). The effectiveness of this approximated model is verified in [26, 28]. Specifically, the linearized DistFlow equations are formulated as follows,

$$P_{i+1}=P_i - p_i, \forall i \in N \tag{2a}$$

$$Q_{i+1}=Q_i - q_i, \forall i \in N \tag{2b}$$

$$V_{i+1}=V_i - \frac{r_{i+1}P_{i+1}+x_{i+1}Q_{i+1}}{V_0}, \forall i \in N \tag{2c}$$

$$p_i=p_i^d - p_i^g, \forall i \in N \tag{2d}$$

$$q_i=q_i^d - q_i^g, \forall i \in N \tag{2e}$$

2.3. PV Hosting Capacity Enhancement via Optimal SVC Planning

According to the linearized DistFlow equations (2a)-(2e), the voltage magnitude of node $i+1$ can be expressed as (2c). Thus, the voltage increment ΔV between can be formulated as $\Delta V = V_i - V_{i+1} = \frac{r_{i+1}P_{i+1}+x_{i+1}Q_{i+1}}{V_0}$. With the PV power penetration increase in the node i, the inverse active power flow P_{i+1} increases. Therefore, voltage increment ΔV increases, which may cause overvoltage problem. However, SVC has the capability of offsetting the voltage rise via reactive power consumption. Specifically, SVC can absorb the reactive power to increase the reactive power flow Q_{i+1}, hence, the voltage increment ΔV decreases. Therefore, SVC is helpful in enhancing the PV hosting capacity.

2.4. Mathematical Formulation of SVC Planning Problem

Definition: PV hosting capacity is defined as the maximum total PV capacity that a DN can accommodate without violating operational constraints, especially node voltage constraints.

In this subsection, a novel stochastic planning of SVC is proposed to maximize PV hosting capacity of the DN. In particular, a two-stage stochastic programming model is formulated considering the uncertainties of load demand and PV output. The detailed planning model are described as follows.

2.4.1. Objective Function

We consider two objectives in our formulation. One is to maximize the PV hosting capacity as shown by Eq. (3a), and the other is to minimize SVC planning cost consisting of investment cost and operation cost as shown by Eq. (3b).

$$\text{Max} \sum_m E_m^{PV} \tag{3a}$$

$$\text{Min} \sum_i \eta C_F^{SVC} a_i^{SVC} + \sum_s p_s \sum_t \sum_i C_V^{SVC} a_i^{SVC} \left| q_{its}^{SVC} \right| \tag{3b}$$

where $\eta = \dfrac{ir(1+ir)^y}{365[(1+ir)^y - 1]}$ represents the daily recovery factor, ir is the interest rate of SVC device, and y is the planning horizon.

To deal with the above two objectives simultaneously, we construct a multi-objective function by formulating a weighted sum function as below,

$$\underset{\psi_1, \psi_2}{\text{Min}} \; -w^{PV} \sum_m E_m^{PV} + w^{SVC} (\sum_i \eta C_F^{SVC} a_i^{SVC} + \sum_s p_s \sum_t \sum_i C_V^{SVC} a_i^{SVC} \tilde{q}_{its}^{SVC}) + C^{Penalty} \sum_s p_s \sum_t \sum_i (\overline{s}_{its} + \underline{s}_{its}) \tag{4}$$

where w^{PV} and w^{SVC} are weighting factors, and $w^{PV} + w^{SVC} = 1$. Different weighting factors will result in different tradeoff between the PV hosting capacity (the first line) and SVC planning cost (the second line), as shown in the case studies. In practice, these factors are adjustable depending on the preference of distribution system planners; $\tilde{q}_{its}^{SVC} := \left| q_{its}^{SVC} \right|$; $\psi_1 = \{a_i^{SVC}, Q_i^{SVC}, E_m^{PV}\}$ and $\psi_2 = \{P_{its}, Q_{its}, V_{its}, q_{its}^{SVC}\}$ denote the collection of the first-stage variables and the collection of the second-stage variables, respectively. Note that the third line of objective function (4) represents the penalty cost, which is imposed to avoid the occurrence of voltage violations. Specifically, we introduce two non-negative slack variables $\overline{s}_{its}$ and $\underline{s}_{its}$ to represent the overvoltage and undervoltage violations in the second stage, respectively. If these two variables turn out to be positive, it means that E_m^{PV} obtained from the first stage does not truly evaluate the PV hosting capacity. Hence, it will be revised until the slack variables $\overline{s}_{its}$ and $\underline{s}_{its}$ both converge to zero.

2.4.2. Constraints

The constraints are classified into first-stage constraints and second-stage constraints, where the first-stage constraints are given as,

a) PV Hosting Capacity Limit

$$E_m^{PV} \geq 0, \forall m \in N^{PV}(i) \tag{5a}$$

where (5a) represents that the PV hosting capacity is non-negative.

b) SVC Installation Limit

$$0 \leq Q_i^{SVC} \leq \overline{Q}_i^{SVC}, \quad \forall i \in N \tag{5b}$$

$$\sum_i a_i^{SVC} \leq N_{inv}^{SVC}, \forall i \in N \tag{5c}$$

where (5b) denotes the SVC installation capacity limit in which the upper bound represents the maximum available installation capacity of SVC in practical application. And (5c) describes that the total SVC installation number cannot exceed a predefined number considering the limit of the total capital cost.

The second-stage constraints are given as,

a) Power Flow Constraints

$$P_{i+1ts} = P_{its} + p_{mts}^{PV} - p_{its}^{d}, \forall i \in N, \forall m \in N^{PV}(i), \forall t \in T, \forall s \in S \tag{6a}$$

$$\text{where } p_{mts}^{PV} = \xi_{ts}^{PV} E_m^{PV}$$

$$Q_{i+1ts} = Q_{its} + q_{its}^{SVC} - q_{its}^{d}, \forall i \in N, \forall t \in T, \forall s \in S \tag{6b}$$

$$V_{i+1ts} = V_{its} - \frac{r_{i+1} P_{i+1ts} + x_{i+1} Q_{i+1ts}}{V_0}, \forall i \in N, \forall t \in T, \forall s \in S \tag{6c}$$

$$P_{i+1ts} \le \overline{P}_i, \forall i \in N, \forall t \in T, \forall s \in S \tag{6d}$$

$$Q_{i+1ts} \le \overline{Q}_i, \forall i \in N, \forall t \in T, \forall s \in S \tag{6e}$$

where (6a)-(6c) represent linearized DistFlow equations. Specially, (6a) and (6b) describes the active power flow and reactive power flow. To capture the uncertainty of PV output, we define the PV output factor $\xi_{ts}^{PV} \in [0,1]$ so that PV power p_{mts}^{PV} generated by distributed PV generator allocated to node m at time t in scenario s is $\xi_{ts}^{PV} E_m^{PV}$. (6c) describes the voltage transmit along the branch. (6d) and (6c) give the active and reactive power flow limits, respectively.

b) Voltage Magnitude Constraints

$$\underline{V}_i - \underline{s}_{its} \le V_{its} \le \overline{V}_i + \overline{s}_{its}, \forall i \in N, \forall t \in T, \forall s \in S \tag{6f}$$

$$\overline{s}_{its} \ge 0, \underline{s}_{its} \ge 0, \forall i \in N, \forall t \in T, \forall s \in S \tag{6g}$$

where (6f) shows the relaxed voltage constraints with two slack variables $\overline{s}_{its}$ and $\underline{s}_{its}$, and (6g) shows that these slack variables are non-negative.

c) SVC Operation Constraints

$$-a_i^{SVC} Q_i^{SVC} \le q_{its}^{SVC} \le a_i^{SVC} Q_i^{SVC}, \forall i \in N, \forall t \in T, \forall s \in S \tag{6h}$$

$$\tilde{q}_{its}^{SVC} \ge q_{its}^{SVC}, \forall i \in N, \forall t \in T, \forall s \in S \tag{6i}$$

$$\tilde{q}_{its}^{SVC} \ge -q_{its}^{SVC}, \forall i \in N, \forall t \in T, \forall s \in S \tag{6j}$$

where (6h) imposes limit on the reactive power support of SVC. (6i) and (6j) are used to convert the term $\left|q_{its}^{SVC}\right|$ to $\tilde{q}_{its}^{SVC}$. Note that there is a bilinear term $a_i^{SVC} Q_i^{SVC}$ in constraint (6e), which renders the problem nonconvex . Hence, we introduce an auxiliary variable z_i^{SVC} to replace $a_i^{SVC}Q_i^{SVC}$ with four additional linear inequalities as shown by (7a)-(7b).

$$-a_i^{SVC}\overline{Q}_i^{SVC} + z_i^{SVC} \le 0, \forall i \in N \tag{7a}$$

$$a_i^{SVC}\underline{Q}_i^{SVC} - z_i^{SVC} \le 0, \forall i \in N \tag{7b}$$

$$-a_i^{SVC}\underline{Q}_i^{SVC} + z_i^{SVC} \le Q_i^{SVC} - \underline{Q}_i^{SVC}, \forall i \in N \tag{7c}$$

$$a_i^{SVC}\overline{Q}_i^{SVC} - z_i^{SVC} \le -Q_i^{SVC} + \overline{Q}_i^{SVC}, \forall i \in N \tag{7d}$$

By doing so, the nonlinearity is eliminated. Thus, (6e) is equivalently rewritten as (8).

$$-z_i^{SVC} \le q_{its}^{SVC} \le z_i^{SVC}, \forall i \in N, \forall t \in T, \forall s \in S \tag{8}$$

3. Solution Methodology

In this section, we propose a solution method based on Benders decomposition to solve the proposed two-stage stochastic planning problem. Usually, the proposed stochastic planning problem is intractable because of numerous scenarios and time coupling objective. Thus, this problem cannot be directly solved by the commercial solvers, which is further demonstrated in the case studies. In this respect, we develop a solution method based on Benders decomposition to solve this planning problem. Generally, Benders decomposition is used to reduce the problem complexity by decomposing the original problem into a master problem and a subproblem. In addition, Benders cuts are generated and added to the master problem to build a link between the master problem and the subproblem. As aforementioned, our proposed problem is a two-stage problem and thus is proper to apply Benders decomposition to solve it. The first stage of our problem corresponds to the master problem and the second stage problem corresponds to the subproblem. Moreover, we can decouple the coupled constraints and objective across the time horizon and uncertainty scenarios by further decomposing the second stage problem into multiple subproblems. Each subproblem is only associated with one time period and one scenario. Therefore, our proposed method can significantly improve the computational efficiency.

3.1. Subproblem

The subproblem for each scenario s and each time period t is given as,

$$Z_{ts}^{sub(v)} := \underset{\psi^{sp}}{\mathrm{Min}}\ w^{SVC}\sum_i C_V^{SVC} a_i^{SVC(v)} \tilde{q}_{its}^{SVC(v)} + C^{Penalty}\sum_i (\overline{s}_{its}^{(v)} + \underline{s}_{its}^{(v)}) \tag{9a}$$

$$\text{s.t. (6a)-(6d), (6f)-(6i), (8)} \tag{9b}$$

$$E_m^{PV(v)} = E_m^{PV,fix} : \varphi_{mts}^{PV(v)}, \forall m \in N^{PV}(i) \tag{9c}$$

$$z_i^{SVC(v)} = z_i^{SVC,fix} : \varphi_{its}^{SVC(v)}, \forall i \in N \tag{9d}$$

where v denotes the iteration index of Benders decomposition. $Z_{ts}^{sub(v)}$ denotes the optimal value of subproblem (9).

The decision variables of (9) is given by

$$\psi^{sp} = \{Z_{ts}^{sub(v)}, E_m^{PV(v)}, z_i^{SVC(v)}, P_{its}^{(v)}, Q_{its}^{(v)}, Q_i^{SVC(v)}, a_i^{SVC(v)}, q_{its}^{SVC(v)}, \tilde{q}_{its}^{SVC(v)}, V_{its}^{(v)}, \overline{s}_{its}^{(v)}, \underline{s}_{its}^{(v)}, \varphi_{mts}^{PV(v)}, \varphi_{its}^{SVC(v)}\}$$

The objective function (9a) consists of SVC operation cost and penalty cost for voltage violations. (9b) summarizes the second-stage constraints. $E_m^{PV(v)}$ and $z_i^{SVC(v)}$ are fixed in this subproblem as shown by (9c) and (9d), where $E_m^{PV,fix}$ and $z_i^{SVC,fix}$ are first stage decision variables obtained from the master problem. After solving all the subproblems, we can obtain an upper bound $Z_{upper}^{(v)}$ to the optimal value of the original problem (4)-(8) as follows,

$$Z_{upper}^{(v)} = \sum_s p_s \sum_t Z_{ts}^{sub(v)} - w^{PV} \sum_m E_m^{PV,fix} + w^{SVC} \sum_i \eta C_F^{SVC} a_i^{SVC,fix} \tag{10}$$

Dual variables $\varphi_{mts}^{PV(v)}$ and $\varphi_{its}^{SVC(v)}$ of first-stage variables are used to calculate the sensitivities for generating Benders cuts. These sensitivities can be obtained as follows,

$$\varphi_m^{PV(v)} = \sum_s p_s \sum_t \varphi_{mts}^{PV(v)}, \forall m \in N^{pv}(i), \forall t \in T, \forall s \in S \tag{11a}$$

$$\varphi_i^{SVC(v)} = \sum_s p_s \sum_t \varphi_{its}^{SVC(v)}, \forall i \in N, \forall t \in T, \forall s \in S \tag{11b}$$

3.2. Master Problem

The formulation of Benders master problem is given as,

$$Z_{lower}^{(v)} := \underset{\psi^{mp}}{\text{Min}} \; \lambda^{(v)} - w^{PV} \sum_m E_m^{PV(v)} + w^{SVC} \sum_i \eta C_F^{SVC} a_i^{SVC(v)} \tag{12a}$$

$$\text{s.t. (5a)-(5c), (7a)-(7b)} \tag{12b}$$

$$\lambda^{(v)} \geq \sum_s p_s \sum_t Z_{ts}^{Sub(k)} + \sum_m \varphi_m^{PV(k)} (E_m^{PV(v)} - E_m^{PV(k)}) + \sum_i \varphi_i^{SVC(k)} (z_i^{SVC(v)} - z_i^{SVC(k)}) \quad k = 1, 2, ..., v-1 \tag{12c}$$

$$\lambda^{(v)} \geq \lambda^{down} \tag{12d}$$

$$\lambda^{(v)} - w^{PV} \sum_m E_m^{PV(v)} + w^{SVC} \sum_i \eta C_F^{SVC} a_i^{SVC(v)} \leq Z^{opt} \tag{12e}$$

The decision variables of (12) is given by

$$\psi^{mp} = \{Z_{lower}^{(v)}, E_m^{PV(v)}, z_i^{SVC(v)}, Q_i^{SVC(v)}, a_i^{SVC(v)}, \lambda^{(v)}\}$$

The master problem (12) is mixed-integer linear. $Z_{lower}^{(v)}$ is a lower bound of the original problem (4)-(8) since master problem (12) relaxes the second-stage constraints. (12b) summarizes first-stage constraints. (12c) describes the Benders cut, linking the master problem and the subproblem. (12d) introduces a lower bound λ^{down} for Benders cut $\lambda^{(v)}$ to accelerate the convergence. (12e) guarantees that the objective value $Z_{lower}^{(v)}$ is lower or equal to the minimum upper bound Z^{opt} obtained from the subproblems.

3.3 Benders Decomposition Algorithm Procedure

The proposed bilevel Benders decomposition algorithm for solving the proposed two-stage stochastic SVC planning model is shown as Algorithm1. The convergence is guaranteed until the upper bound meets the lower bound according to [29].

Algorithm 1 Benders Decomposition Algorithm

Step 1. Initialization: Set the iteration index $v=1$. Set the initial upper bound $Z_{upper}^{(v)}=\infty$ and lower bound $Z_{lower}^{(v)}=-\infty$. Set the convergence tolerance ε. Initialize the first-stage variables, $E_m^{PV(0)}$ and $z_i^{SVC(0)}$. Set $E_m^{PV,fix}=E_m^{PV(0)}$ and $z_i^{SVC,fix}=z_i^{SVC(0)}$.

Step 2. Iteration: Solve the subproblem (9) for each time period and each uncertainty scenario. Obtain the upper bound $Z_{upper}^{(v)}$ according to (10).

Step 3. Minimum upper bound update: If $Z_{upper}^{(v)} \le Z^{opt}$, update the global solution $Z^{opt}=Z_{upper}^{(v)}$.

Step 4. Convergence check: If $|Z_{upper}^{(v)}-Z_{lower}^{(v)}| \le \varepsilon$, then terminate with the optimal solution. Otherwise, calculate the sensitivities by equations (11a) and (11b) to build the next Benders cut. Then, set $v \leftarrow v+1$.

Step 5. Solve master problem: Solve the master problem (12), calculate $Z_{lower}^{(v)}$ and update the values of $E_m^{PV,fix}$ and $z_i^{SVC,fix}$. Then go back to the step 2 and continue.

4. Case Studies

4.1 Implementation on IEEE 37-node Distribution system

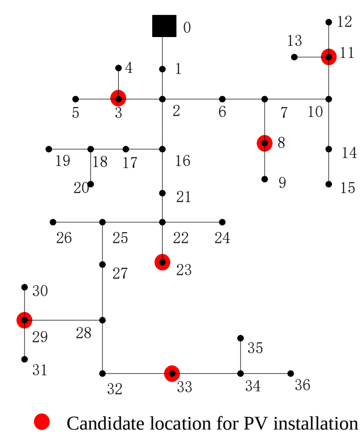


Fig. 3 Modified IEEE 37-node test distribution system.

Fig. 3 shows the IEEE 37-node test distribution system. We assume there are six locations are suitable for PV installation, namely nodes 3, 8, 11, 23, 29 and 33. Details about the test system can be found in [30]. Per-unit value is used in case studies. The base values of power and voltage are set as 1 MVA and 12.66 kV, respectively. We consider a 10-year planning horizon. One hundred representative scenarios are generated to characterize the uncertainties. In this paper, as an example, one combination of weighting factor is selected to show the performance of our proposed model and algorithm, i.e. $w^{PV}=0.5$ and $w^{SVC}=0.5$.

4.1.1. Convergence Performance

Fig. 4 shows the convergence of the proposed Benders decomposition based algorithm. Table 1 compares the computational efficiency of two approaches. One is to directly solve the original problem (4)-(8) using a commercial solver GUROBI [31] on the platform of CVX [32], denoted as CVX-GUROBI. The other is to solve the original problem (4)-(8) using our proposed Benders decomposition based algorithm via the same platform and solver, demoted as CVX_BD-GUROBI. Table I demonstrates that the original problem (4)-(8) cannot be directly solved by the commercial solver GUROBI due to great computational complexity. However, our proposed algorithm is efficient in solving the same problem.

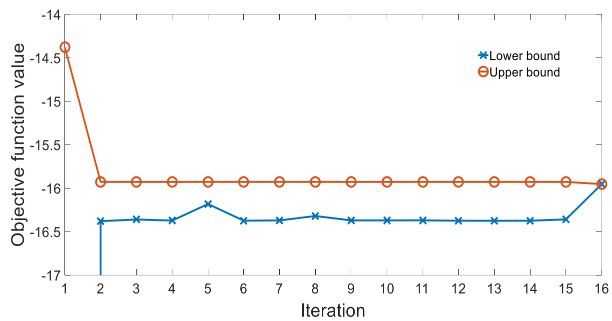


Fig. 4 Convergence of the proposed Benders decomposition based algorithm.

Table 1
Comparison on Computation Time of Solving the Proposed SVC Planning Problem (under 100 scenarios).

CVX-GUROBI	CVX_BD-GUROBI	
[sec.]	[sec.]	[iterations]
NA	8211	16

4.1.2. Optimal Results of PV Hosting Capacity and SVC Planning

Table 2 lists PV hosting capacity for the selected sites. The total PV hosting capacity of the 37-node test distribution system is 0.491 p.u.. The corresponding SVC planning decisions are shown in Table 3.

4.1.3. Performance of SVC Planning Result on PV Hosting Capacity

Fig. 5 depicts PV hosting capacity of two cases: 1) the base case without SVC installation; 2) the case with stochastic optimal SVC planning. It can be observed that the PV hosting capacity of case 2 is significantly higher than that of the case 1, which demonstrates the effectiveness of the stochastic optimal SVC planning in improving the PV hosting capacity. Fig. 6 shows the voltage profiles of node 13 at 1:00 pm under three cases: 1) base case (without installation of both PV and SVC), 2) case with PV installation as the result in Table 2 but without SVC installation, 3) case with PV installation as the result in Table II and SVC installation as the result in Table 3. It can be observed that voltage magnitudes of some nodes exceed the upper bound in case 2, but all overvoltage violations are alleviated after the optimal SVC planning as shown by the curve of case 3.

Table 2
Results of PV Hosting Capacity in 37-node Test System.

Candidate Location (Node)	PV Size (p.u.)	Candidate Location (Node)	PV Size (p.u.)
3	0.121	23	0.109
8	0.082	29	0.036
11	0.062	33	0.081

Table 3

Results of SVC Planning in 37-node Test System.

Location (Node)	SVC Size (p.u.)	Location (Node)	SVC Size (p.u.)
3	0.050	23	0.050
4	0.032	24	0.050
7	0.042	26	0.041
8	0.050	29	0.050
9	0.027	33	0.050
10	0.022	34	0.031
11	0.050	-	-

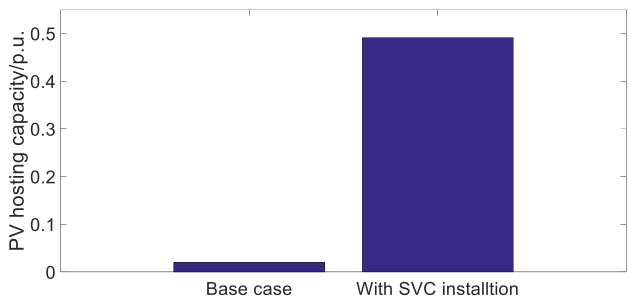


Fig. 5 Comparison on PV hosting capacity.

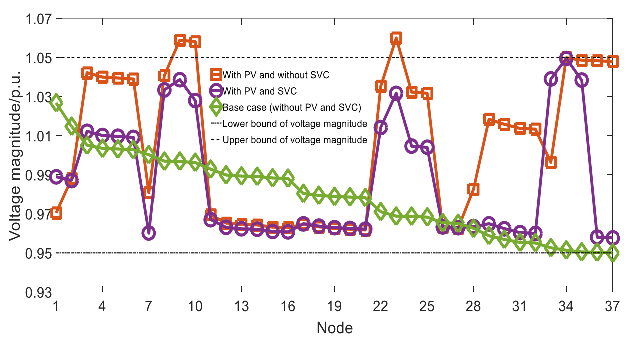
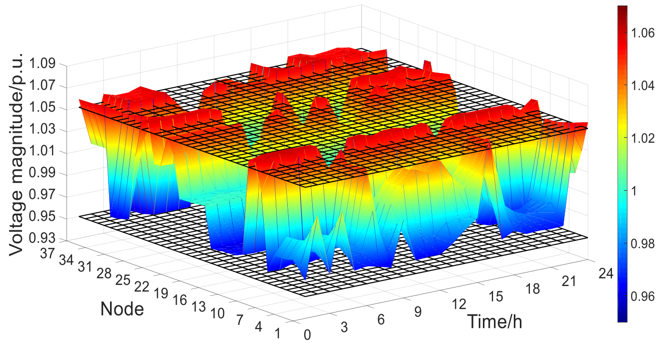


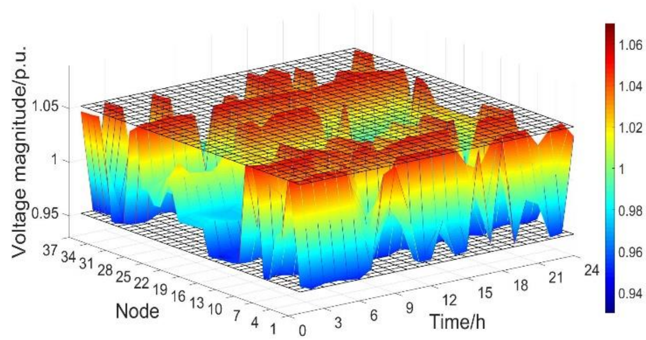
Fig. 6 Comparison on voltage profiles.

4.1.4 Compared with Deterministic Scheme

The deterministic SVC planning scheme is used as benchmark here. The formulation of the deterministic optimal SVC planning problem is similar to (4)-(8) but with only one scenario. The sites under the deterministic SVC planning scheme are node 6, 7, 8, 9, 10, 11, 23, 24, 26, 29, 33 and 34. The corresponding sizes are 0.02, 0.035, 0.05, 0.05, 0.019, 0.05, 0.05, 0.043, 0.05, 0.05, 0.05 and 0.043 p.u., respectively. We also obtain the PV hosing capacity under the deterministic scheme, i.e. 0.1, 0.07, 0.04, 0.1, 0.03 and 0.06 p.u. for nodes 3, 8, 11, 23, 29, and 33, respectively. We define the critical scenario as the scenario with the highest PV power output factor ξ_{ts}^{PV} and lowest load demand level, and compare the performance of the stochastic result and the deterministic result under this critical scenario. Fig. 7 (a) and (b) show the voltage profiles of the deterministic scheme and the stochastic scheme under the critical scenario, respectively. The overvoltage violations are observed in Fig. 7 (a), while there is no voltage violations in Fig. 7 (b). The reason is that stochastic scheme considers more scenarios and thus it is more comprehensive and robust in dealing with the uncertainties.



(a)



(b)

Fig. 7 Comparison result of (a) the deterministic scheme and (b) the stochastic scheme under the critical scenario.

4.1.5. Optimal Tradeoff Curve

Fig. 8 shows the optimal tradeoff curve between the PV hosting capacity and SVC planning cost. We can see that the PV hosting capacity increases linearly with the raise of the SVC planning cost until the cost reaches $7,500. Then the increasing rate decreases gradually to zero, which means the PV hosting capacity becomes insensitive to the additional planning cost when the total cost exceeds $1,7500.

4.1.6. Sensitivity Analysis

In order to investigate the impact of SVC installation capacity and number on the PV hosting capacity, two sensitivity

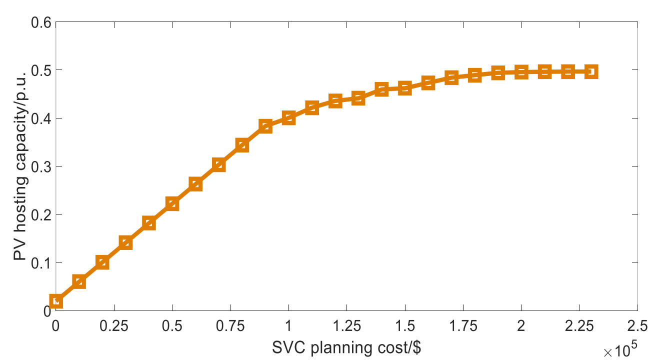
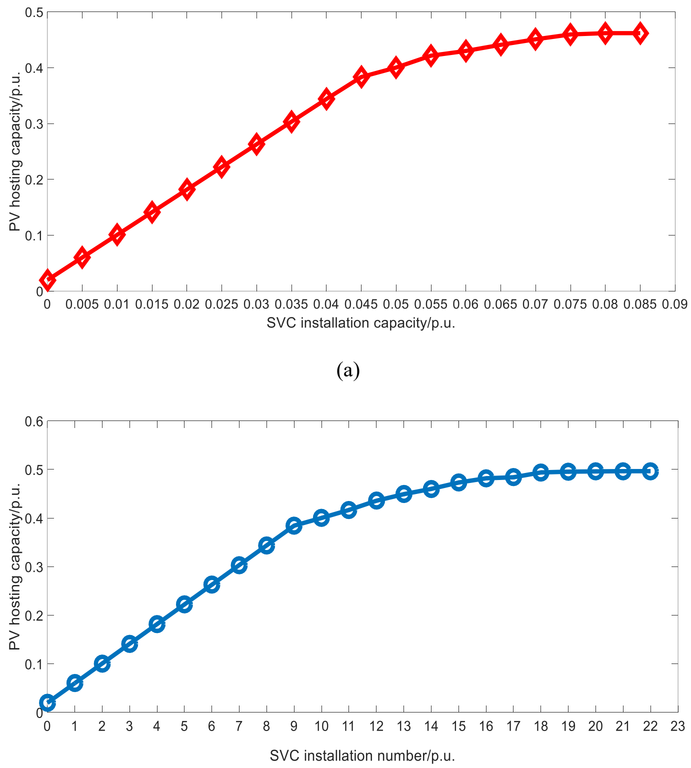


Fig. 8 Optimal tradeoff curve between the PV hosting capacity and SVC planning cost.

analyses are conducted. Fig. 9 (a) illustrates the impact of SVC installation capacity on the PV hosting capacity with the SVC installation number fixed at 10 and Fig. 9 (b) illustrates the impact of SVC installation number on the PV hosting capacity with the installation capacity of each SVC being 0.05p.u.. It can be observed from Fig. 9 that the PV hosting capacity improves almost linearly with the increase of SVC installation capacity/number until the installation capacity reaches 0.045 p.u. and the installation number reaches 9. Then the PV hosing capacity becomes less sensitive and eventually insensitive to the increase of SVC installation capacity/number. This is because larger PV power penetration may lead to the DN line overload. Under such circumstance, DN line capacity expansion planning can be suggested if the PV hosting capacity is too small to be accepted by DN planners.



(a)

(b)

Fig. 9 Impact of (a) SVC installation capacity (with same installation number;10) and (b) SVC installation number (with same installation capacity: 0.05p.u.) on PV hosting capacity under the expected scenario.

4.2. Implementation on IEEE 123-node Distribution system

The proposed model is also tested on the modified IEEE 123-node distribution system as shown in Fig. 10. The detailed parameters can be found in [30]. In this case, base values of power and voltage, uncertainty scenarios and weighting factors are same as those in the 37-bus case. Twelve candidate locations are selected for PV installation, i.e. nodes 5, 23, 31, 34, 45, 58, 62, 77, 84, 93, 109 and 118. Results of PV hosting capacity and SVC planning are listed in Table IV and Table V, respectively. The total PV hosting capacity of the 123-node test distribution system is 2.459 p.u.. The daily voltage magnitudes of the modified 123-node distribution system under the critical scenario are shown in Fig. 11. Similar to the 37-bus case, the voltage magnitudes are ensured within the allowable ranges.

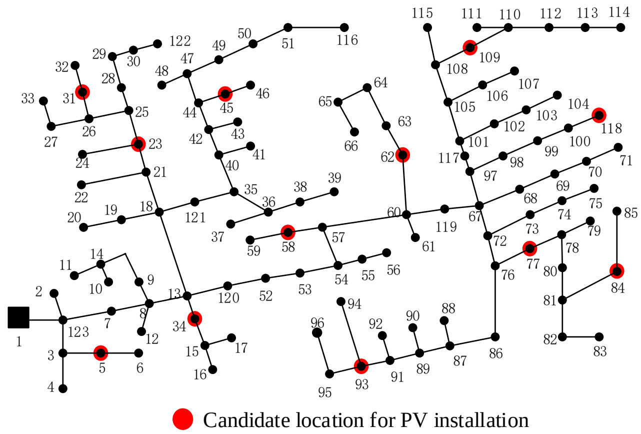


Fig. 10 The modified IEEE 123-node test distribution system.

Table 4

Results of PV Hosting Capacity In 123-node System.

Candidate Location (Node)	PV Size (p.u.)	Candidate Location (Node)	PV Size (p.u.)
5	0.209	62	0.093
23	0.224	77	0.102
31	0.214	84	0.315
34	0.181	93	0.243
45	0.212	109	0.102
58	0.339	118	0.225

Table 5

Results of SVC Planning In 123-node System.

Location (Node)	SVC Size (p.u.)	Location (Node)	SVC Size (p.u.)	Location (Node)	SVC Size (p.u.)
5	0.050	37	0.009	84	0.050
6	0.034	45	0.050	85	0.036
22	0.015	47	0.040	93	0.050
23	0.050	57	0.043	94	0.048
25	0.003	58	0.050	109	0.050
30	0.043	59	0.050	117	0.008
31	0.050	62	0.050	118	0.050
33	0.044	77	0.050	119	0.028
34	0.050	83	0.050	-	-

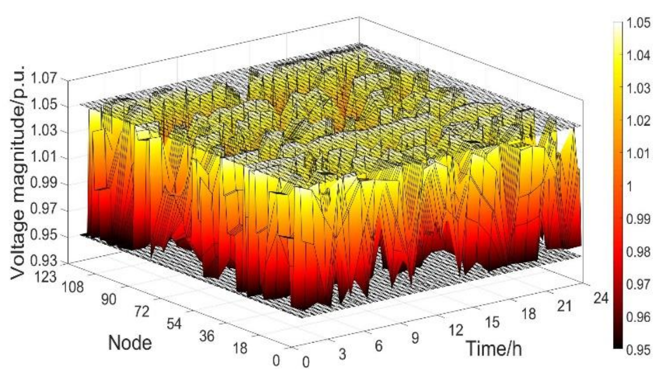


Fig. 11 The daily voltage magnitudes of the modified 123-node distribution system under the critical scenario.

6. CONCLUSION

This paper presents a novel two-stage stochastic SVC planning model to enhance PV hosting capacity considering uncertainties of load demand and PV output. In the first stage, the SVC planning decisions and the corresponding PV hosting capacity are determined. In the second stage, the feasibility of the first stage decisions is evaluated under multiple uncertainty scenarios to ensure no voltage violations. To improve the computational efficiency, an efficient solution method based on Benders decomposition is developed to solve this two-stage problem. Numerical results on modified IEEE 37-node and 123-node distribution systems verify the effectiveness of the proposed model and solution method.

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