IET Research Journals

Submission Template for IET Research Journal Papers

ISSN 1751-8644
doi: 0000000000
www.ietdl.org

Distributed Noise-resilient Economic Dispatch Strategy for Islanded Microgrids

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Abstract: Economic dispatch (ED) plays an important role in safeguarding secure and economic operation of the microgrid (MG). However, the communication links among distributed generators (DGs) may practically be corrupted by additive noise, resulting in erroneous deviations from ED commands or even blackouts. To deal with such issues, this paper proposes a distributed noise-resilient ED strategy for the islanded MG based on the consensus algorithm. Because of consensus-based implementation, the proposed ED strategy is implemented in a fully distributed manner, which enables the peer-to-peer communication among DGs without the necessity of a central controller. In addition, the proposed ED strategy involves post-iterate averaging technique, to further enhance its convergence under additive communication noise. In this way, different from the existing ED strategies, the proposed ED strategy is fully distributed and resilient to the communication noise. Further, the effectiveness of the proposed ED strategy is evaluated on an example islanded MG consisting of renewable energy and DGs, where the communication links are corrupted by different levels of additive noise. Finally, in order to fully take account of the uncertainty and stochastic nature of different operation scenarios, the Monte Carlo simulations are carried out, of which the simulation results demonstrate that the proposed ED strategy is superior over the existing ED strategies, in terms of the convergence property under additive communication noise, especially in the case of large noise.

Nomenclature

Sets	
V	Set of agents
E	Set of communication links
Parameters	
n	Number of DGs
a_i, b_i, c_i	Generation cost coefficients of ith DG
$P_i^{\min}, P_i^{\max}$	Lower and upper limits of power generation of ith DG
λ, $\overline{u}$, $\underline{u}$	Lagrange multipliers
T	Sample time
k	Discrete time step
g_{ij}	Link control gain between ith Agent and jth Agent
Variables	
P_i	Active power output of ith DG
P_{RG}	Power output of photovoltaic and wind generation
P_{D}	Total load demand
λ_i	Incremental cost of ith DG
λ_i^*	Optimal incremental cost of ith DG
$X[k]$	Incremental costs at kth iteration
$\bar{X}[k]$	Average incremental costs at kth iteration
$\Delta X[k]$	Changes of incremental costs at kth iteration
$p_{ij}[k]$	Power transfer from ith DG to jth DG
$d_{ij}[k]$	Additive noise in the communication link between ith DG and jth DG
$\hat{x}_{ij}[k]$	Corrupted incremental cost of DG_j acquired by DG_i
μ	Step size

1 Introduction

1.1 Motivation and incitement

The microgrid (MG) is generally defined as a cluster of distributed generators (DGs), renewable energy and loads, which is considered as a promising solution to reduce fossil fuel emissions and pollution. The MG can operate both in grid-connected and islanded modes, and the control of an islanded MG is more challenging than that of a grid-connected MG, due to the intermittency of renewable energy and low inertia nature of the islanded MG [1]. For an islanded MG, an essential concern is how to minimize the total generation cost of the MG while satisfying both equality and inequality constraints, i.e., the economic dispatch (ED) problem. Note that the total generation cost of the MG is minimized when the incremental costs of DGs are equal, i.e., the equal incremental cost criterion, where the incremental cost is the derivative of DG cost function with respect to the output power [2, 3].

Conventionally, the ED problem was solved by centralized strategies, all of which require bidirectional communication links between a central controller and all generation units. Therefore, the centralized ED strategies may possibly encounter several challenges. First, the central controller may be vulnerable to targeted cyber and physical attacks, which result in the whole-system failure; second, it is costly to establish point-to-point communication links when the scale of the MG increases; third, the centralized ED strategies may raise information privacy concern. In contrast to the centralized ED strategies, the distributed ED strategies are simply based on the local information, which reduces the communication complexity and obviates the need of a central controller [4–9]. Therefore, in consideration of increasing number of DGs and price-responsive demands, the distributed ED strategies are more suitable for solving ED problem.

1.2 Literature Review

In the recent years, the distributed ED strategies have attracted much attention. Among them, the consensus-based ED strategies are most popular, because the agreement on incremental costs can be ensured by iterative update rules of the consensus algorithm, i.e., the equal incremental cost criterion can be satisfied [10–14]. Note that the ED sometimes involves the coordination between the generation and demand side of the MG [15–17]. Therefore, in order to minimize the total generation cost of DGs while maximizing the benefit of demand side, the ED and demand side management problems were solved in an integrated manner [18–24]. Moreover, in order to expedite the convergence speed of consensus-based ED strategies, the distributed ED strategies based on finite-time consensus algorithm were developed, to guarantee fast convergence to the optimal solutions [25–27].

As discussed above, the existing distributed ED strategies solve the ED problem in a fully distributed manner, without the necessity of a central controller and point-to-point communication. However, the existing distributed ED strategies assume that the communication in the MG is noise-free. In practice, due to the noise generated by electronic components and environmental causes, the communication links are possibly corrupted by additive noise, which inevitably degrades the performance and stability of the distributed ED strategies [28–31]. In other words, the desired performances of the distributed ED strategies are not warranted in the presence of communication noise. Therefore, it is desirable to develop the distributed noise-resilient ED strategy to mitigate the impact of communication noise.

So far, some interest has been focused on the distributed noise-resilient control strategies for the MG. For instance, a noise-resilient control technique for voltage and frequency synchronization in the MG is proposed [28, 29]. Furthermore, reference [32] developed a distributed noise-resilient secondary voltage and frequency control as well as power sharing strategy for the MG, which is fully independent of MG parameters and noise type. However, to the best of our knowledge, the distributed noise-resilient ED strategy is rarely investigated in the existing literatures.

1.3 Contribution and Organization

In such a context, a distributed noise-resilient ED strategy is proposed in this paper, to minimize the total generation cost of the MG. First, a consensus-based ED strategy is proposed, which is implemented in a fully distributed manner. Later, in order to mitigate the communication noise impact, the post-iterate averaging technique is designed and incorporated into the consensus-based ED strategy, to develop the proposed distributed noise-resilient ED strategy, and its convergence is guaranteed under the additive communication noise, even if the communication noise is large. Moreover, in consideration of uncertain ED solutions following variant load demands and power outputs of non-dispatchable DGs, a large number of Monte Carlo simulations are carried out in this paper, to further validate its performance against the communication noise.

Compared to the existing ED strategies, the salient features of the proposed ED strategy are summarized as follows. *First*, it is fully distributed such that it simply requires sparse communication links and local information exchange. *Second*, it is resilient to the communication noise, i.e., it is able to mitigate the impact of the communication noise. *Third*, it is implemented in a straightforward way such that the classical consensus-based ED strategy can be easily extended to the proposed noise-resilient ED strategy, therefore making more sense for real applications. *Fourth*, it does not require a detailed model of the MG, and it is fully independent of MG parameters.

The rest of the paper is organized as follows. The ED problem of the islanded MG is briefly introduced in Section 2. In Section 3, the distributed noise-resilient ED strategy is developed, to minimize the total generation cost of the islanded MG. In Section 4, the structure and parameters of the islanded MG for simulations are introduced and listed, respectively. Later, in Section 5, six simulation cases are implemented on an example islanded MG, to evaluate the performance of the proposed ED strategy, and then the simulation results are analyzed and discussed. Finally, Section 6 concludes the paper.

2 Economic Dispatch Problem

This section briefly reviews the formulation of the ED problem as well as its classical solution, which are based on the Lagrangian method and equal incremental cost criterion.

2.1 Control Objective

We consider an islanded MG consisting of a photovoltaic (PV) generation, a wind generation (WG) and n dispatchable DGs. The photovoltaic and wind generation are considered as non-dispatchable DGs, both of which operate in maximum power point tracking (MPPT) control mode. It is assumed that the non-dispatchable photovoltaic and wind generation are free, while the ith dispatchable DG has a quadratic generation cost function of $C_i(P_i)$. Moreover, the transmission loss is about 5% of the total load demand [20]. Combining the power balance and power generation constraints, the ED problem of the islanded MG can be formulated as follows

$$\min \sum_{i=1}^{n} C_i(P_i) = \min \sum_{i=1}^{n} (a_i \cdot P_i^2 + b_i \cdot P_i + c_i), \tag{1}$$

$$\sum_{i=1}^{n} P_i + P_{\text{RG}} = P_D + P_{\text{loss}} = (1 + 5\%) \cdot P_D, \tag{2}$$

$$P_i^{\min} \leq P_i \leq P_i^{\max}, \tag{3}$$

where P_i is the active power output of DG_i, $C_i(P_i)$ is the generation cost function of DG_i, a_i, b_i, c_i are cost coefficients, P_{RG} is the total active power output of photovoltaic and wind generation, P_D is the total active power load demand in the MG, P_{loss} is the transmission loss, $P_i^{\min}$, $P_i^{\max}$ are lower and upper limits of active power generation of DG_i, respectively.

2.2 The Equal Incremental Cost Criterion

The ED problem formulated in (1) can be solved by the Lagrangian method, and the corresponding Lagrangian function takes the following forms [10, 21]

$$\begin{aligned} &L(P_1, P_2, \cdots, P_n) \\ &= \sum_{i=1}^{n} C_i(P_i) + \lambda \cdot (P_D + P_{\text{loss}} - \sum_{i=1}^{n} P_i - P_{\text{RG}}) \\ &\quad + \sum_{i=1}^{n} \overline{u_i} \cdot (P_i - P_i^{\max}) + \sum_{i=1}^{n} \underline{u_i} \cdot (P_i^{\min} - P_i), \end{aligned} \tag{4}$$

where λ, $\overline{u_i}$, and $\underline{u_i}$ are Lagrange multipliers.

In terms of first order optimality conditions, the solution to the ED problem can be obtained, which is the *equal incremental cost criterion*, and it takes the following forms

$$\begin{cases} \lambda_i = \frac{\partial C_i(P_i)}{\partial P_i} = 2a_i \cdot P_i + b_i = \lambda^* & P_i^{\min} < P_i < P_i^{\max}, \\ \lambda_i = \frac{\partial C_i(P_i)}{\partial P_i} = 2a_i \cdot P_i + b_i < \lambda^* & P_i = P_i^{\max}, \\ \lambda_i = \frac{\partial C_i(P_i)}{\partial P_i} = 2a_i \cdot P_i + b_i > \lambda^* & P_i = P_i^{\min}, \end{cases} \tag{5}$$

where λ_i and λ^* are incremental cost and optimal incremental cost of DG_i, respectively.

The equal incremental cost criterion denotes that the total active power generation cost of the MG is minimized, when the equal incremental cost criterion is satisfied. Moreover, in this case, DGs that operate within power generation limits have the equal incremental cost of $\frac{\partial C_i(P_i)}{\partial P_i}$, and DGs have the incremental costs of $\frac{\partial C_i(P_i)}{\partial P_i}|_{P_i=P_i^{\max}}$ or $\frac{\partial C_i(P_i)}{\partial P_i}|_{P_i=P_i^{\min}}$, when operating on their upper or lower generation limits, respectively.

3 Distributed Noise-resilient Economic Dispatch Strategy

In this section, a consensus-based ED strategy is proposed first. Next, the noise-resilient ED strategy is developed considering additive noise in the commutation links among DGs.

3.1 Implementation Architecture

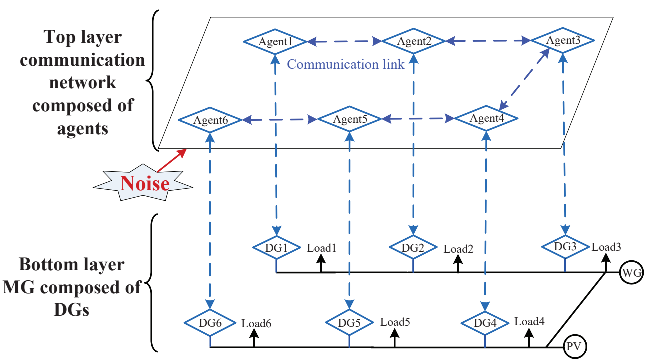


Fig. 1: The two-layer control model for the islanded MG

In this paper, a two-layer control model based on the multi-agent system (MAS) for the islanded MG is established first, where the top layer is a communication network composed of agents, while the bottom layer is an MG composed of DGs, PV and wind generation systems, as shown in Fig. 1. Next, each agent collects the present states of the corresponding DG, and then exchanges the acquired information with its neighboring agents. After the acquired information is processed according to the proposed ED strategy, agents adjust the power outputs of the corresponding DGs, in order to minimize the total generation cost of the MG.

In practice, some mature communication technologies are used for information exchange in the MG, e.g., TCP/IP, optical fiber and power line carrier communication. Note that the communication links among DGs are possibly corrupted by additive noise, which is generated by electro-magnetic interference, electronic components, radio signals, smart sensors and environmental causes, etc. In this paper, we consider additive noise in all the communication links among DGs, and the additive communication noise is statistically modeled as Gaussian noise. Without loss of generality, the communication noise is considered to be zero mean Gaussian noise, and the non-zero mean Gaussian noise can be transformed into zero mean Gaussian noise [28].

3.2 Post-iterate Averaging for Convergence Under Communication Noise

In terms of the graph theory, the communication network in the two-layer control model is a directed graph $G(V, E)$ with n agents and r communication links, where V is the set of agents, and E is the set of communication links. For a selected sample time T for information updates, the consensus algorithm is performed at discrete-time steps $kT, k = 1, 2, 3, \cdots$.

Regarding the physical meaning of ED in the islanded MG, the incremental costs of DGs can be considered as the state values of agents. In consequence, the incremental costs of DGs at the kth and $(k+1)$th iteration are denoted by $X[k] = [x_1[k], \cdots, x_i[k], \cdots, x_n[k]]'$, $X[k+1] = [x_1[k+1], \cdots, x_i[k+1], \cdots, x_n[k+1]]'$, respectively, and the prime means transpose of the vector [33]. Therefore, the incremental cost update process can be expressed as follows

$$X[k+1] = X[k] + \Delta X[k], \tag{6}$$

where $\Delta X[k] = [\Delta x_1[k], \cdots, \Delta x_i[k], \cdots, \Delta x_n[k]]'$ are changes of the incremental costs.

Based on the definition of incremental cost in (5), the change of the incremental cost of DG$_i$ is determined by the power transfer $p_{ij}[k]$ from DG$_i$ to DG$_j$ at the kth iteration

$$\begin{aligned} \Delta x_i[k] &= -\sum_{(i,j)\in G} \Delta x_{ij}[k] \\ &= -\sum_{(i,j)\in G} (2a_i \cdot p_{ij}[k] + b_i). \end{aligned} \tag{7}$$

where $\Delta x_{ij}[k]$ is the change of incremental cost of DG$_i$ caused by the power transfer from DG$_i$ to DG$_j$ at the kth iteration.

On the other hand, due to the additive noise $d_{ij}[k]$ in the communication link r_{ij}, DG$_i$ receives the information of incremental cost of DG$_j$ with the additive noise

$$\hat{x}_{ij}[k] = x_j[k] + d_{ij}[k], \tag{8}$$

where $\hat{x}_{ij}[k]$ is the corrupted incremental cost of DG$_j$ acquired by DG$_i$.

Let $\hat{X}[k]$ and $D[k]$ be the r-dimensional vectors that contain all $\hat{x}_{ij}[k]$ and $d_{ij}[k]$ in a selected order, respectively. Later, (8) can be written as

$$\hat{X}[k] = H_1 X[k] + D[k], \tag{9}$$

where H_1 is an $r \times n$ matrix whose rows are elementary vectors such that if the lth element of $\hat{X}[k]$ is $\hat{x}_{ij}[k]$ then lth row in H_1 is the row vector of all zeros except for a "1" at the jth position.

Let $\delta[k]$ be an r-dimensional vector containing all $\delta_{ij}[k]$ in the same order as $\hat{X}[k]$, where $\delta_{ij}[k] = x_i[k] - \hat{x}_{ij}[k]$ is the estimated difference between the incremental costs of DG$_i$ and DG$_j$ at the kth iteration. Subsequently, in terms of (8), $\delta[k]$ can be expressed as follows

$$\delta[k] = H_2 X[k] - \hat{X}[k] = HX[k] - D[k], \tag{10}$$

where H_2 is an $r \times n$ matrix whose rows are elementary vectors such that if the lth element of $\hat{X}[k]$ is $\hat{x}_{ij}[k]$ then the lth row in H_2 is the row vector of all zeros except for a "1" at the ith position, and $H = H_2 - H_1$.

In terms of the communication network shown in Fig. 1, the information of incremental cost is exchanged locally. Therefore, the information of $\delta_{ij}[k]$ is simply available to DG$_i$ and DG$_j$, and we have $\Delta x_{ij}[k] = \mu[k] g_{ij}[k] \delta_{ij}[k]$, where g_{ij} is the link control gain, and $\mu[k]$ is the recursive step size [34]. Let G be an $r \times r$ diagonal matrix that has link control gain g_{ij} as its diagonal element, and all link control gains are positive, $g_{ij} > 0$. Later, the changes of the increments cost can be expressed as follows

$$\Delta X[k] = -\mu[k] H' G \delta[k]. \tag{11}$$

Later, in terms of (10) and (11), $\Delta X[k]$ is equivalent to

$$\begin{aligned} \Delta X[k] &= -\mu[k] H' G (HX[k] - D[k]) \\ &= -\mu[k](H' G H X[k] - H' G D[k]) \\ &= \mu[k](MX[k] + WD[k]), \end{aligned} \tag{12}$$

where $M = -H'GH$, $W = H'G$.

Furthermore, according to (6) and (12), we have

$$X[k+1] = X[k] + \mu[k][MX[k] + WD[k]]. \tag{13}$$

Thereafter, we assume the following conditions.

Assumption 1: (1) All link gains are positive, $g_{ij} > 0$. (2) G is strongly connected.

Assumption 2: (1) The step size satisfies the following conditions: $\mu[k] \geq 0, \mu[k] \longrightarrow 0$ as $k \longrightarrow \infty$. (2) The noise $D[k]$ is a stationary ϕ-mixing sequence such that $E(D[k]) = 0$, $E(|D[k]|^{2+\Delta}) < \infty$ for some $\Delta > 0$, and for any positive integer m, the mixing measure $\tilde{\phi_n}$ satisfies

$$\sum_{i=0}^{\infty} \tilde{\phi}_i^{\Delta/(1+\Delta)} < \infty, \tag{14}$$

where

$$\begin{cases} \tilde{\phi}_k = \sup_{A \in F^{k+m}} E^{(1+\Delta)/(1+\Delta)} |P(A|F_m) - P(A)|^{(2+\Delta)/(1+\Delta)}, \\ F_k = \sigma\{D[i]; i < k\}, \quad F^k = \sigma\{D[i]; i \geq k\}. \end{cases} \tag{15}$$

Remark 1: Note that a square matrix $\tilde{Q} = \tilde{q}_{ij}$ is a generator of a continuous-time Markov chain if $\tilde{q}_{ij} \geq 0$ for all $i \neq j$ and $\sum_{j=1}^{j=n} \tilde{q}_{ij} = 0$. Moreover, a generator or the associated continuous-time Markov chain is irreducible if the system of equations

$$\begin{cases} \nu \tilde{Q} = 0, \\ \nu e = 1, \end{cases} \tag{16}$$

has a unique solution, where $e = [1, 1, \cdots, 1]'$, $\nu = [\nu_1, \nu_2, \cdots, \nu_n]$. Further, under the *Assumption 1*, we have (1) M has rank $n-1$ and is negative semi-definite. (2) M is a generator of a continuous-time Markov chain, and M is irreducible [34, 35].

Remark 2: Under the *Assumption 2*, and in terms of *Remark 1*, we have

$$\lim_{k \longrightarrow \infty} X[k] = \frac{ee'}{n} X[0], \tag{17}$$

where $X[0] = [x_1[0], \cdots, x_i[0], \cdots, x_n[0]]'$ are initial incremental costs of DGs.

The proof of *Remark 2* uses the similar ideas as in [34] and hence is omitted here. In terms of *Remark 2*, we know that if DGs update their incremental costs according to the update rules formulated in (13), then the incremental costs will converge to the average of the initial values, namely, satisfying equal incremental cost criterion.

Finally, the *consensus-based ED strategy* is formulated as follows

$$\begin{cases} X[k+1] = X[k] + \mu[k][MX[k] + WD[k]], \\ M = -H'GH, \ W = H'G, \\ H = H_2 - H_1. \end{cases} \tag{18}$$

In term of the consensus-based ED strategy and two-layer control model in Fig. 1, agents calculate the incremental costs of corresponding DGs first, and then agents exchange the information of incremental costs with their neighboring agents. Next, agents calculate the set points of active power outputs for DGs at the next iteration in terms of (5) and (18). Later, agents send the set points to the corresponding DGs, and then the power outputs of DGs are regulated to achieve the consensus of the incremental costs, satisfying the equal incremental cost criterion. Namely, the total generation cost of the islanded MG is minimized.

Remark 3: Note that the consensus-based ED strategy is within the framework of standard stochastic approximation method, and we have a limit ordinary differential equation as follows

$$\dot{\chi} = M\chi. \tag{19}$$

Let $M\chi = 0$, the equilibria of (19) can be obtained. Sine M is a generator of a continuous-time Markov chain, the equilibria of (19) constitute the set $Z = \{z \in \mathbb{R}\}$, and $z = c\mathbb{1}$ for any real number $c \in \mathbb{R}$. In other words, the equilibria are the set of r-dimensional vectors spanned by the vector $\mathbb{1}$. When $c = 0$, we get the equilibrium point 0, therefore, Z is the set of consensus. Note that the convergence of the consensus-based ED strategy is closely related to the associated ordinary differential equation of (19). In terms of the ordinary differential equation methods elaborated in [36], we take a continuous-time interpolation $\chi^{\mu}(t) = \chi_n$ for $t = [\mu n, \mu n + \mu]$ and study the limit dynamics through the trajectories of differential equations whose stationary points belong to Z. Recall that a set S is said to be locally stable in the sense of Lyapunov if for each $\delta > 0$ there is a $\delta_1 > 0$ such that all trajectories starting in the δ_1-neighborhood $N_{\delta_1}(S)$ of S never leave the δ-neighborhood $N_{\delta}(S)$ of S. If the trajectories ultimately go to S, then S is said to be asymptotically stable in the sense of Lyapunov. If this holds for all initial conditions, then the asymptotic stability is said to be global [34, 37, 38].

The consensus-based ED strategy has desirable convergence property when the communication is noise-free, which will be demonstrated in the simulation results. However, the communication links among DGs may possibly be corrupted by additive noise, which results in the erroneous deviations from ED commands or even blackouts. To deal with such situations, the post-iterate averaging technique is specially designed and incorporated into the consensus-based ED strategy, to develop *the distributed noise-resilient ED strategy*, which is able to mitigate the impact of communication noise, and guarantee its convergence under communication noise.

The proposed distributed noise-resilient ED strategy consists of two stages. The first stage is a coarse approximation of the optimal incremental cost, which is obtained using the consensus-based ED strategy formulated in (18). Later, the second stage provides a refinement by averaging the incremental costs obtained in the first stage, which will mitigate the communication noise impact, resulting in smaller variances, and the second stage is given by

$$\begin{aligned} \bar{X}[k+1] &= \frac{1}{k+1} \sum_{j=1}^{j=k+1} X[j] \\ &= \frac{1}{k+1} [\sum_{j=1}^{j=k} x[j] + X[k+1]] \\ &= \bar{X}[k] - \frac{1}{k+1} \bar{X}[k] + \frac{1}{k+1} X[k+1] \\ &= \bar{X}[k] + \frac{1}{k+1} [X[k+1] - \bar{X}[k]], \end{aligned} \tag{20}$$

where $\bar{X}[k]$ and $\bar{X}[k+1]$ are average incremental costs obtained at the kth and $(k+1)$th iteration, respectively.

In terms of (18) and (20), *the distributed noise-resilient ED strategy* is obtained as follows

$$\begin{cases} X[k+1] = X[k] + \mu[k][MX[k] + WD[k]], \\ \bar{X}[k+1] = \bar{X}[k] + \frac{1}{k+1} [X[k+1] - \bar{X}[k]], \end{cases} \tag{21}$$

where $\bar{X}[k+1] = [\bar{x}_1[k+1], \cdots, \bar{x}_i[k+1], \cdots, \bar{x}_n[k+1]]'$ are reference incremental costs for DGs at the $(k+1)$th iteration.

The proposed distributed noise-resilient ED strategy belongs to the class of iterate averaging. The idea is that we first obtain a rough approximation by using a relatively large step size $\mu[k] = 1/k^{\alpha}$, so that the iterates will be fast for reaching the vicinity of the optimal solution. In the second stage, the approximation is refined by taking an iterate averaging, which will mitigate the noise impact resulting in smaller variances.

Following this idea, the $\mu[k]$ should be a sequence of step sizes satisfying

$$\mu[k] \geq 0, \lim_{k \longrightarrow \infty} \mu[k] = 0. \tag{22}$$

In this paper, we take $\mu[k] = 1/k^{\alpha}$ for $0.5 < \alpha < 1$. In terms of the analysis in reference [39], such a control strategy is able to achieve the convergence under communication noise.

4 Microgrid System Architecture

In this section, the performance of the distributed noise-resilient ED strategy is tested on an islanded MG shown in Fig. 1. The cost coefficients and power limits of dispatchable DGs are listed in Table 1 [25]. The sample time for the ED strategy is set to 25 ms when the simulations are carried out, and the MG system works in a balanced state initially. Additionally, the data of load, solar irradiance and wind speed are randomly generated in this paper. In other words, there are no constraints for the data, so the readers can easily reproduce the simulation results to validate the performance of the proposed ED strategy in communication noise mitigation.

Table 1 Cost coefficients of quadratic generation cost functions

DG	a	b	c	$P^{\min}$	$P^{\max}$
DG_1	0.074	3.17	62	4.2 kW	20 kW
DG_2	0.094	1.22	51	10 kW	45 kW
DG_3	0.105	2.53	78	3.8 kW	40 kW
DG_4	0.082	4.02	42	5.4 kW	45 kW
DG_5	0.078	3.41	31	5 kW	30 kW
DG_6	0.066	3.52	26	5 kW	45 kW

It should be emphasized that the topology of the MAS communication network is independent of the MG structure. In other words, it is not required that the topology of the communication network is identical to that of the MG. Therefore, many possible communication networks can be considered for an MG with meshed or radial structure. In this paper, the communication network shown in Fig. 1 is designed for the MG with radial structure. Suppose the order for the communication links is selected as r_{12}, r_{21}, r_{23}, r_{32}, r_{34}, r_{43}, r_{45}, r_{54}, r_{56}, r_{65}, in terms of the definitions of H_1 and H_2, we have

$$H_1 = \begin{bmatrix} 0&1&0&0&0&0\\ 1&0&0&0&0&0\\ 0&0&1&0&0&0\\ 0&1&0&0&0&0\\ 0&0&0&1&0&0\\ 0&0&1&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&1&0&0\\ 0&0&0&0&0&1\\ 0&0&0&0&1&0 \end{bmatrix}, H_2 = \begin{bmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{bmatrix}. \quad (23)$$

Next, we choose the link control gain matrix as
$G = \mathrm{diag}[0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2]$.
Later, we have

$$H = H_2 - H_1 = \begin{bmatrix} 1&-1&0&0&0&0\\ -1&1&0&0&0&0\\ 0&1&-1&0&0&0\\ 0&-1&1&0&0&0\\ 0&0&1&-1&0&0\\ 0&0&-1&1&0&0\\ 0&0&0&1&-1&0\\ 0&0&0&-1&1&0\\ 0&0&0&0&1&-1\\ 0&0&0&0&-1&1 \end{bmatrix}, \quad (24)$$

$$M = -H'GH = \begin{bmatrix} -0.4&0.4&0&0&0&0\\ 0.4&-0.8&0.4&0&0&0\\ 0&0.4&-0.8&0.4&0&0\\ 0&0&0.4&-0.8&0.4&0\\ 0&0&0&0.4&-0.8&0.4\\ 0&0&0&0&0.4&-0.4 \end{bmatrix}, \quad (25)$$

$$W = H'G = \begin{bmatrix} 0.2&-0.2&0&0&0&0&0&0&0&0\\ -0.2&0.2&0.2&-0.2&0&0&0&0&0&0\\ 0&0&-0.2&0.2&0.2&-0.2&0&0&0&0\\ 0&0&0&0&-0.2&0.2&0.2&-0.2&0&0\\ 0&0&0&0&0&0&-0.2&0.2&0.2&-0.2\\ 0&0&0&0&0&0&0&0&-0.2&0.2 \end{bmatrix}. \quad (26)$$

Further, in terms of (25), (26), and the distributed noise-resilient ED strategy formulated in (21), the reference incremental costs for dispatchable DGs at next iteration can be calculated. After that, in terms of (5), the set points of active power outputs for DGs can be obtained and implemented, to minimize the total generation cost of the islanded MG.

5 Numerical Results

To evaluate the performance of the proposed distributed noise-resilient ED strategy, six simulation cases are carried out in this section using the MATLAB. First, the basic case is carried out to validate the feasibility of the proposed noise-resilient ED strategy in solving ED problem without the communication noise. Second, in order to investigate the impact of the communication noise on ED strategy, the classical consensus-based ED strategy is implemented in context of communication noise. Third, the proposed distributed noise-resilient ED strategy is tested under the same communication noise in the previous case, to validate its feasibility for mitigating the noise impact. The fourth focuses on the impacts of different communication network topologies on the system performance. The fifth investigates the plug and play capability of the proposed ED strategy. Finally, the Monte Carlo simulations considering the stochastic load demands and power outputs of non-dispatchable DGs are carried out with 10,000 trials, to further validate the robustness of the proposed noise-resilient ED strategy against the uncertain operation scenarios with communication noise.

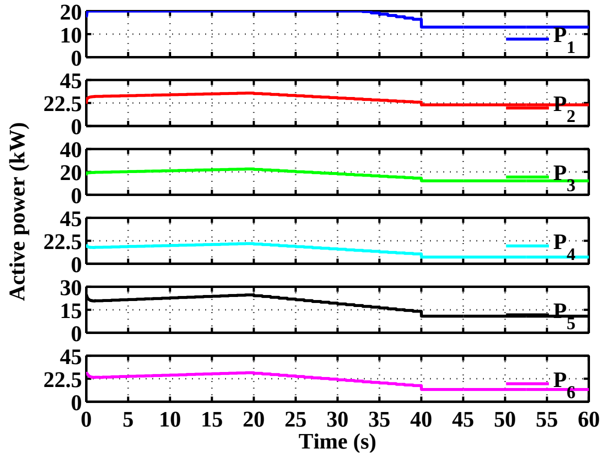
5.1 The Performance of the Proposed Noise-resilient ED Strategy Without the Communication Noise

In order to validate the feasibility of the proposed noise-resilient ED strategy in solving ED problem, the proposed noise-resilient ED strategy is implemented in this case with the noise-free communication. Moreover, the load demands and power outputs of non-dispatchable PV and wind generation fluctuate over time. For instance, from $t = 0$ s to $t = 40$ s, the active power outputs of non-dispatchable DGs change over time, and the total load demand decreases dramatically at $t = 40$ s. As a result, the active power outputs of all dispatchable DGs change simultaneously, fulfilling the generation-demand equality constraint in the islaned MG, as shown in Fig. 2(a).

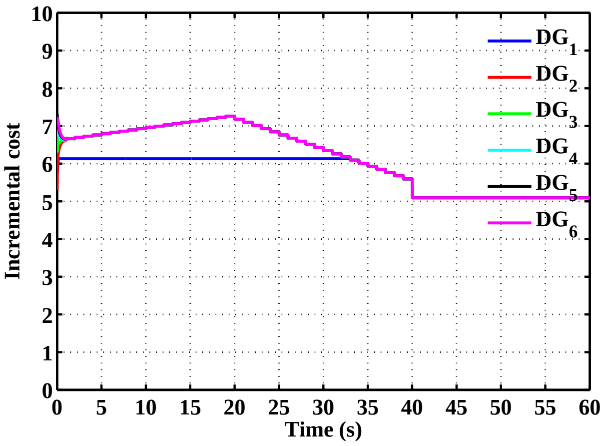
Note that the incremental cost is an increasing function of the active power output, therefore, the incremental costs increase with the rising power outputs of dispatchable DGs from $t = 0$ s to $t = 20$ s, as illustrated in Fig. 2(b). Similarly, due to the decrease in total active power load demand at $t = 40$ s, the incremental costs of all dispatchable DGs decrease from 5.6 to 5.1 simultaneously, as illustrated in Fig. 2(b). Correspondingly, in terms of (5), the active power outputs of DG_1, DG_2, DG_3, DG_4, DG_5 and DG_6 are 13.04 kW, 20.64 kW, 12.24 kW, 6.59 kW, 10.83 kW and 11.97 kW, respectively, which are consistent with the simulation results shown in Fig. 2(a).

On the other hand, the power output of DG_1 reaches its upper limit of 20 kW from $t = 0$ s to $t = 33$ s, therefore, the power generation of DG_1 is fixed at 20 kW. Correspondingly, in terms of (5), the incremental cost of DG_1 is maintained at 6.13 at the same time, which is consistent with the simulation result shown in Fig. 2(b).

Finally, it can be found from Fig. 2(b) that the consensus of incremental costs is ensured by consensus-based ED strategy except



(a) active power outputs of dispatchable DGs



(b) incremental costs of dispatchable DGs

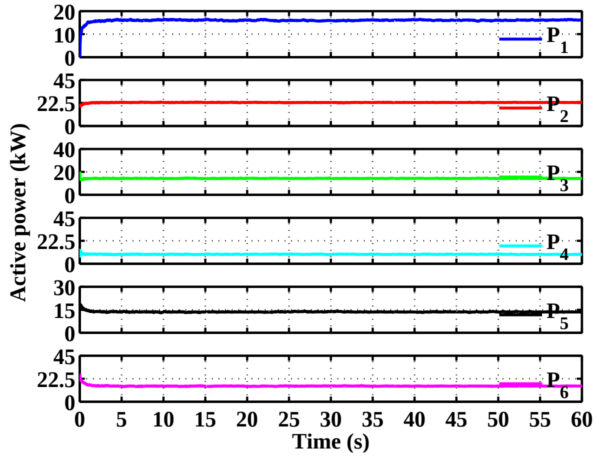
Fig. 2: Simulation results under the fluctuating load demands and non-dispatchable DGs power outputs, when the proposed noise-resilient ED strategy is implemented and the communication is noise-free.

the extreme situation when the power output of DG_1 reaches its upper limit. Therefore, according to the equal incremental cost criterion, the proposed noise-resilient ED strategy minimizes the total generation cost of the islanded MG.

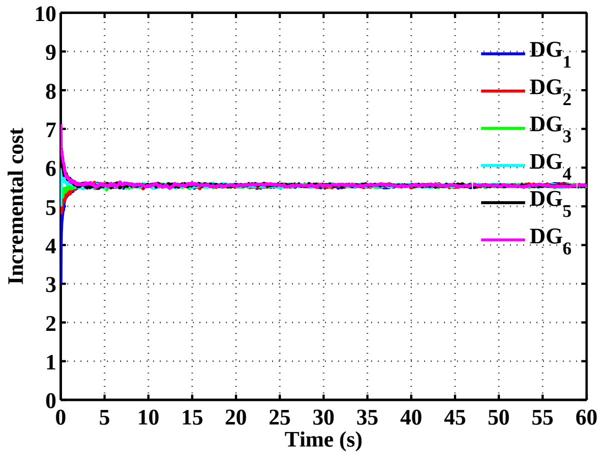
5.2 The Impact of the Communication Noise on Classical Consensus-based ED Strategy

It has been noticed that the proposed noise-resilient ED strategy guarantees the optimal solution to the ED problem without the communication noise. However, in practice, the communication is possibly corrupted by the communication noise, which deteriorates the performance of the ED strategy. In order to investigate the impact of the communication noise, the load demands and power outputs of non-dispacthable DGs are assumed to be constant in this case, and the classical consensus-based ED strategy is implemented when the communication noise is relatively small and large, respectively.

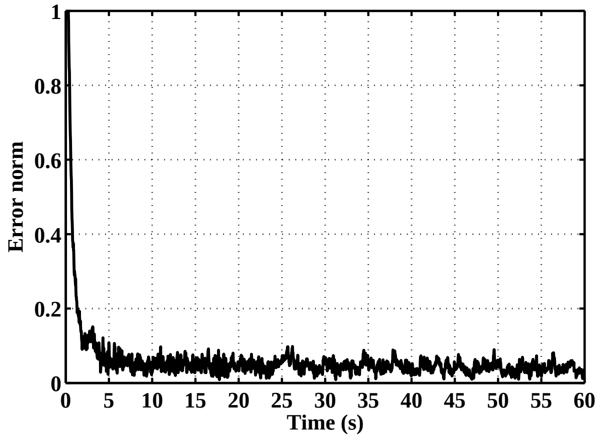
First, the simulation is performed where the communication noise is a sequence of Gaussian noise with zero mean and variance of 0.2 (small noise). Under this situation, the active power outputs and incremental costs of DGs are shown in Fig. 3(a) and Fig. 3(b), respectively. Fig. 3(b) shows how incremental costs whose initial values are different from each other are gradually distributed to the desired consensus. Moreover, the consensus error of the ED strategy



(a) active power outputs of dispatchable DGs



(b) incremental costs of dispatchable DGs



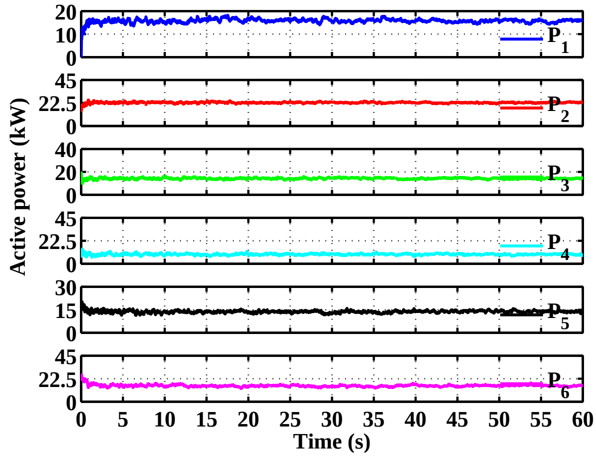
(c) error norm

Fig. 3: Simulation results under the communication noise with zero mean and variance of 0.2 (small noise), when the classical consensus-based ED strategy is implemented.

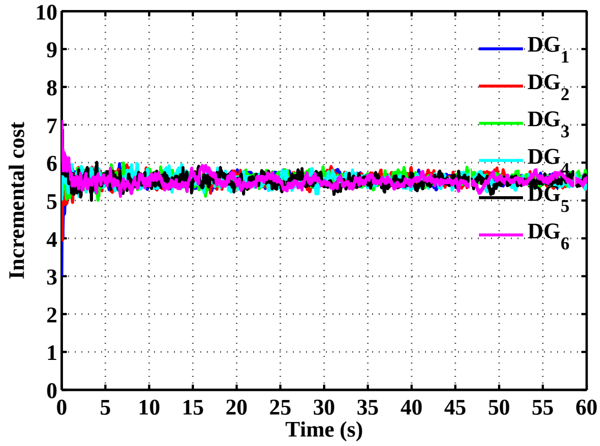
is evaluated using the error norm, which is calculated as follows

$$\text{Error norm} = [(X[k] - \frac{1}{n}\sum_{i=1}^{i=n} x_i[k]\cdot e)'(X[k] - \frac{1}{n}\sum_{i=1}^{i=n} x_i[k]\cdot e)]^{\frac{1}{2}}. \tag{27}$$

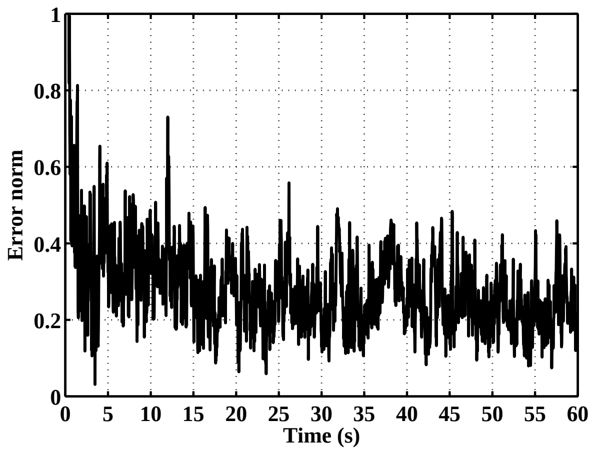
The trajectory of the error norm is plotted in Fig. 3(c). It can be seen from Fig. 3(c) that the error norm converges to zero, implying



(a) active power outputs of dispatchable DGs

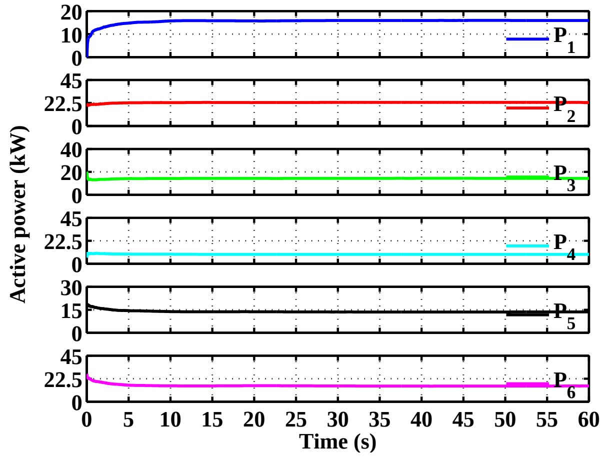


(b) incremental costs of dispatchable DGs

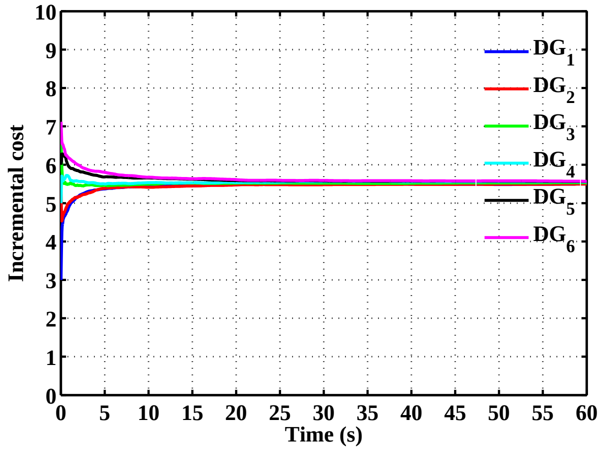


(c) error norm

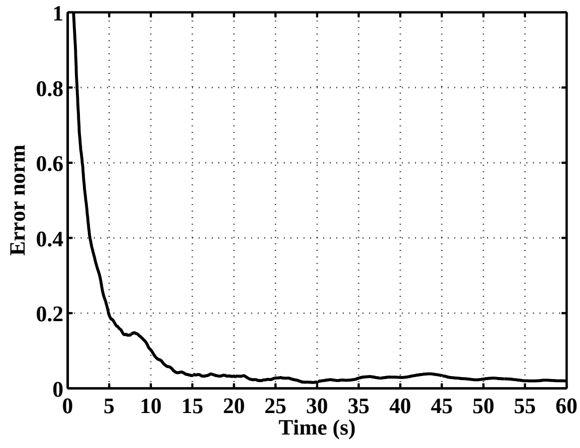
Fig. 4: Simulation results under the large communication noise, when the classical consensus-based ED strategy is implemented.



(a) active power outputs of dispatchable DGs



(b) incremental costs of dispatchable DGs



(c) error norm

Fig. 5: Simulation results under the large communication noise, when the proposed noise-resilient ED strategy is implemented.

that the consensus of the incremental costs is achieved and the total generation cost of the MG is minimized.

However, for the same system, if the communication links are corrupted by the Gaussian noise with zero mean and variances of 1, 1.1 or 1.2, namely, the communication links are corrupted by heterogeneous levels of large noise, the simulation results are shown in Fig. 4, which demonstrate large variations in power outputs and incremental costs of DGs. In other words, the large communication noise results in erroneous deviations from references and malfunction of the consensus-based ED strategy.

5.3 The Performance of the Proposed Noise-resilient ED Strategy Under the Communication Noise

Note that the communication noise results in the malfunction of the classical consensus-based ED strategy, therefore, we propose the noise-resilient ED strategy in this paper. Subsequently, in order to test the performance of noise-resilient ED strategy against the communication noise, the noise-resilient ED strategy is implemented in this case, when the communication links are corrupted by the Gaussian noises with zero mean and variances of 1, 1.1 or 1.2.

The power outputs of DGs are illustrated in Fig. 5(a), and it can be seen from Fig. 5(b) that the incremental costs gradually converge to a consensus. Moreover, the error norm converges to zero, as shown in Fig. 5(c). Compared with the simulation results of classical consensus-based ED strategy under large communication noise in Fig. 4, the noise-resilient ED strategy demonstrates improved convergence with less fluctuations, even if the communication is corrupted by large noise.

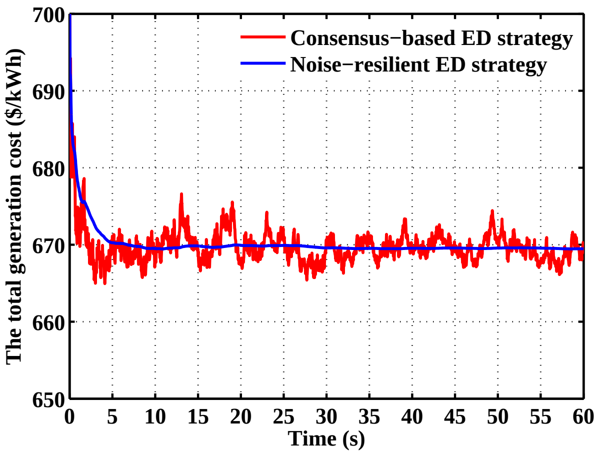


Fig. 6: The total generation cost of the islanded MG with the consensus-based and noise-resilient ED strategies, respectively.

Moreover, the total generation costs of the islanded MG with consensus-based and noise-resilient ED strategies are illustrated in Fig. 6, respectively. It can be found from Fig. 6 that the communication noise results in erroneous deviations from ED references and malfunction of the ED strategy, which indicate non-convergence of the consensus-based ED strategy under the communication noise. In comparison, the noise-resilient ED strategy is able to mitigate the noise impact, and guarantee improved resiliency of the ED strategy to the communication noise.

5.4 The Topologies of Communication Networks versus the Performance of the Proposed Noise-resilient ED Strategy

To test the performance of the proposed noise-resilient ED strategy on different communication networks, a communication network with a different topology $G_1(V, E)$ is designed, as shown in Fig. 7. Further, the simulation results on the new network $G_1(V, E)$ is shown in Fig. 8, when the communication noise is large. Compared with the results on $G(V, E)$ in Case 3 and Fig. 5, it can be found that almost the same results are obtained. This is because the topology of the communication network is independent of the MG structure. Therefore, many possible communication networks can be considered for a give MG structure.

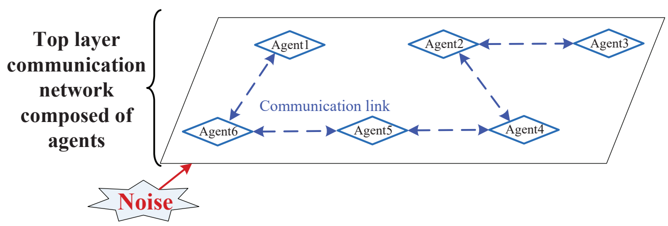
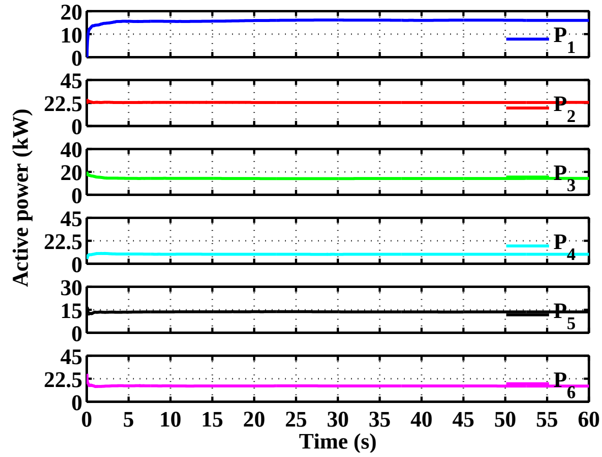
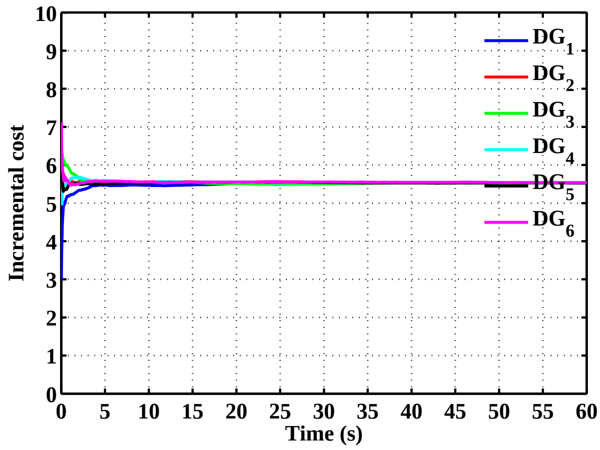


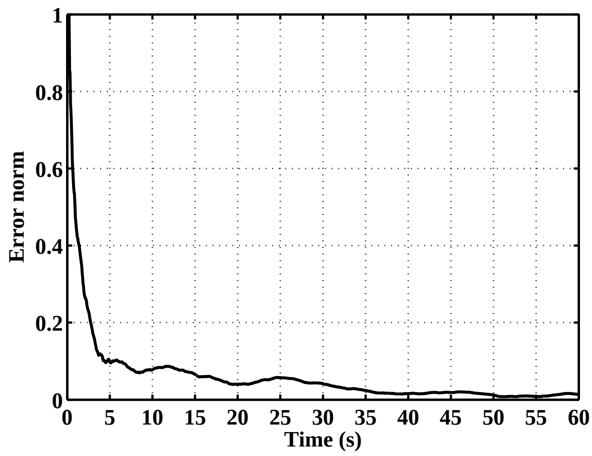
Fig. 7: The topology of a communication network, $G_1(V, E)$.



(a) active power outputs of dispatchable DGs



(b) incremental costs of dispatchable DGs

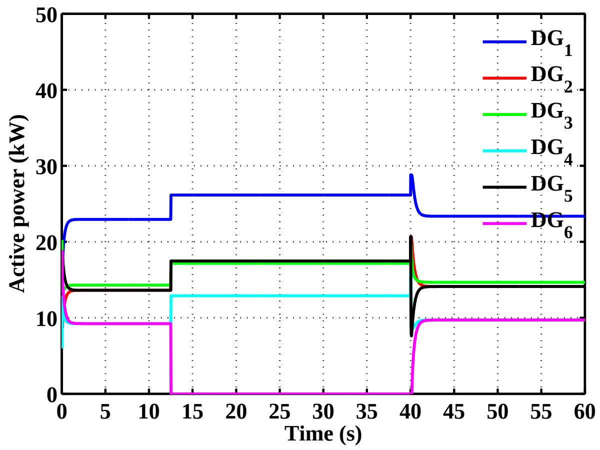


(c) error norm

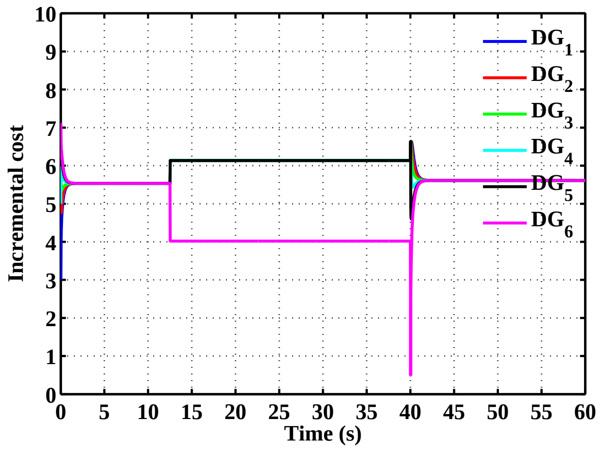
Fig. 8: Simulation results under the large communication noise, when the noise-resilient ED strategy and a different communication network are implemented.

5.5 The Plug and Play Capability of the Proposed Noise-resilient ED Strategy

It has been noticed in previous cases that the proposed ED strategy effectively mitigates the impacts of the communication noise. In practice, the faulted DG will be disconnected from the MG, and the DG will be reconnected to the MG after the maintenance. In order to investigate the plug and play capability of the proposed ED strategy,



(a) active power outputs of dispatchable DGs



(b) incremental costs of dispatchable DGs

Fig. 9: Simulation results under the plug and play operation of DG_6.

the DG_6 is disconnected from the MG and reconnected to the MG at t = 12 s and at t = 24 s, respectively.

When DG_6 is disconnected from the MG at t = 12 s, the remaining DGs have to generate more power to compensate for the amount of power previously generated by DG_6, to maintain the power balance in the islanded MG, as shown in Fig. 9. Thereafter, the DG_6 is reconnected to the MG at t = 24 s. Correspondingly, the other DGs reduce their power outputs to accommodate the reconnection of DG_6, as shown in Fig. 9(a).

Moreover, it can be found in Fig. 9(b) that the proposed ED strategy maintains the consensus of the incremental costs, regardless of the plug and play operation of DG_6. Therefore, the proposed ED strategy has the plug and play capability.

5.6 The Performance of the Proposed Noise-resilient ED Strategy Considering Stochastic Scenarios

In the previous study, only single operation scenario is considered. In practice, the ED strategy has to tackle variant operation scenarios with high uncertainties. Therefore, the Monte Carlo simulations are implemented hereby to consider a series of potential scenarios, to further validate its performance against the communication noise.

To that end, the proposed noise-resilient ED strategy is implemented in this case, and the communication is corrupted by the Gaussian noise with zero mean and variances of 1. Thereafter, we perform Monte Carlo simulations with 10,000 trials, where a ±40% tolerance on load demands and power outputs of non-dispatchable DGs was assumed [40]. The error norm boxplots are shown in Fig. 10, which indicate that the proposed noise-resilient ED strategy reduces the error norm to almost zero in all the probable scenarios. Namely, it ensures that incremental costs converge to a consensus, to minimize the total generation cost of the islanded MG.

Furthermore, the Monte Carlo simulations are implemented with the noise variances of 0.6, 0.9 and 1.2, respectively, and the Monte Carlo simulation terminates when the error norm converges to 0.05. Subsequently, the cumulative distribution function curves of iterations under different levels of noise are plotted in Fig. 11, indicating that the error norm will converge to 0.05 within 1000 iterations with a probability of 100%. Moreover, the larger the noise variance is, the more iterations will perform for the error norm to converge to 0.05.

Finally, the Monte Carlo simulation results in this case further confirm the effectiveness of proposed noise-resilient ED strategy in solving ED problem with highly variable load demands, and power outputs of non-dispatchable DGs.

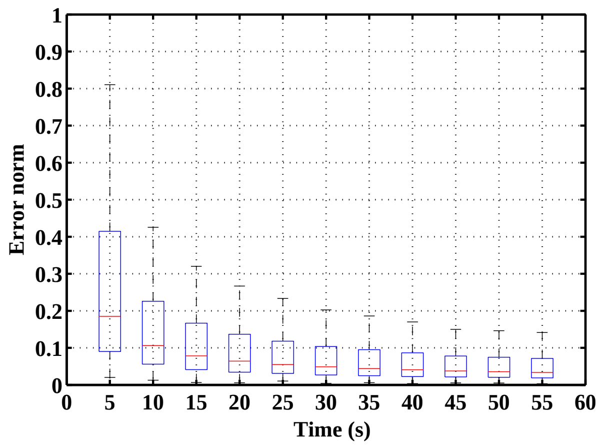


Fig. 10: Error norm boxplots during the Monte Carlo Simulations

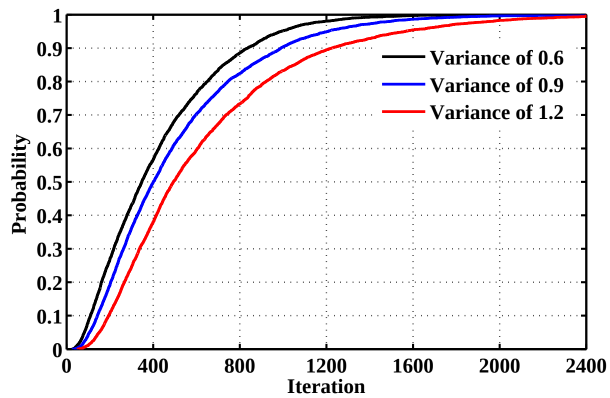


Fig. 11: Cumulative distribution function curves under different levels of communication noise

6 Conclusion

In this paper, a consensus-based noise-resilient ED strategy was developed for the islanded AC MG. The proposed ED strategy is fully distributed such that each DG simply requires the local information and obviates the necessity for a central controller. Thereafter, with the utilization of the post-iterate averaging technique, the convergence of the proposed ED strategy is guaranteed under the additive communication noise, and the convergence and stability of the proposed ED strategy was analyzed and proved. Moreover, the proposed ED strategy is easy to be implemented and offers a straightforward way to mitigate the impacts of the communication noise, by simply deriving the reference power outputs for DGs in terms of the given unified steps.

Different simulation cases are carried out to test the performance of the proposed ED strategy, which show that the total generation cost minimization and communication noise mitigation are obtained. In addition, the proposed ED strategy is not strongly associated with the topologies of communication networks, therefore, there are not many constraints for the communication network design. Moreover, the proposed ED strategy has the plug and play capability, which improves the flexibility and scalability of the control strategy for MG implementation. Finally, the performance of the proposed ED strategy is evaluated under different levels of communication noise using the Monte Carlo simulation method. As demonstrated in the Monte Carlo simulation results, the convergence of the proposed noise-resilient ED strategy to the global optimum, and its robustness against the communication noise are further validated.

It is worth noting that the parametric uncertainties affect the performance of the proposed ED strategy. Therefore, how to improve the robustness of the proposed ED strategy against the parametric uncertainties requires our further research. Moreover, the hardware-in-the-loop simulation of the proposed ED strategy is also a direction of our future work.

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