An Uncertainty involved Optimization Model of Renewable Hosting Capacity Enhancement

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***Abstract*— Renewable hosting capacity normally serves as an important real-time operational index for assessing the capability of accommodating renewables in a distribution network (DN). This paper proposes a two-stage operation model to coordinate energy storage system (ESS) and static var compensator (SVC) to avoid violations of voltage magnitude and line capacity caused by intermittent renewables so as to enhance the renewable hosting capacity. This model comprehensively considers the uncertainties involved in photovoltaic (PV), wind generation and electrical load demand in the first stage and real-time operation states in the second stage. Furthermore, to improve the computational efficiency, a Benders decomposition based solution method is developed to tackle the time-coupling constraints at the first-stage optimization problem. The effectiveness of the proposed model and solution method are demonstrated through case studies on IEEE 33-bus distribution system.**

***Index Terms*-- Renewable hosting capacity, distribution network, a two-stage operation model, uncertainties, Benders decomposition.**

NOMENCLATURE

Sets and Indices

j/J	Index/set of distribution nodes.
t/T	Index/set of time periods.
w/W	Index/set of scenarios.
i	Index of child nodes of the node j.
m	Index of nodes with photovoltaic (PV) energy generation installation.
n	Index of nodes with wind energy generation installation.
b	Index of nodes with SVCs installation.
l	Index of nodes with ESSs installation.
$\varphi(j)$	Set of child nodes of the node j.
$\Omega^{pv}(j)$	Set of child nodes of the node j with PV generation installation.
$\Omega^{wt}(j)$	Set of child nodes of the node j with wind generation installation.
$\Omega^{svc}(j)$	Set of child nodes of the node j with SVCs installation.
$\Omega^{ess}(j)$	Set of child nodes of the node j with ESSs installation.

Variables

P_{jtw}/Q_{jtw}	Active/reactive power at j,t,w.
$P_{jtw}^{qt}/Q_{jtw}^{qt}$	Quadratic term of active/reactive power flow at j,t,w.
V_{jtw}	Node voltage magnitude at j,t,w.
E_m^{pv}	PV hosting capacity allocated to the node m.
E_n^{wt}	Wind power hosting capacity allocated to the node n.
$s_{jtw}^{v,lb}/s_{jtw}^{v,ub}$	Slack variables for voltage violations at j,t,w.
s_{jtw}^{pq}	Slack variable for line capacity and substation capacity of power flow at j,t,w.
P_{lt}^{ess}	Active energy provided by ESS at l,t.
P_{lt}^{ch}/P_{lt}^{dch}	Charge/discharge energy of ESS at l,t.
$\alpha_{lt}^{ch}/\alpha_{lt}^{dch}$	Charge/discharge binary indicators of ESS at l,t.
Q_{btw}^{SVC}	Reactive energy provided by SVC at b,t,w.

Parameters

$c^{Penalty}$	Objective function coefficient associated to the penalty cost for voltage violation and overload.
β_t	Electrical price at t.
$\mu_{tw}^{pv}/\mu_{tw}^{wt}$	PV/wind energy output factor energy output factor (ratio of installed energy generation capacity) at t,w.
η^{ch}/η^{dch}	Charging/Discharging efficiency of ESS.
r_{ij}/x_{ij}	Resistance/Reactance of the DN line ij.
p_{jtw}/q_{jtw}	Active/Reactive load at j,t,w.
$V^{\min}/V^{\max}$	Lower/Upper bound of voltage magnitude.
$Q^{SVC,\min}/Q^{SVC,\max}$	Lower/Upper bound of reactive energy provided by SVC.
LC_{ij}	Capacity of DN line ij.
SC	Capacity of DN substation.

I. INTRODUCTION

Renewable distributed generation (RDG) technology is becoming a promising strategy to address the worldwide energy crisis and environmental pollution problem. Increasing green energy sources, especially photovoltaic (PV) and wind power, are integrated into the distribution networks (DNs) via RDG technology. Nevertheless, some impacts may arise from the widespread use of renewable energy, such as overvoltage, reverse power flow and power loss increase. Moreover, the RDG output disrupts normal operation conditions of DN due to the stochastic nature of renewable energy sources. Therefore, it is quite important to enhance the renewable hosting capacity (RHC) of the DN.

RHC of a DN is defined as the maximum renewable power that a DN is capable to accommodate without operating constraint violations, such as overvoltage and overload. This means that the voltage magnitudes of all network nodes must

This work was supported by Hong Kong RGC Theme Based Research Scheme Grants No. T23-407/13N and T23-701/14N.

remain within the American National Standards Institute (ANSI) Range [1], a limit of $0.95 \le V_{p.u.} \le 1.05$ in steady state. Besides, the network currents must remain within the line and device rated limits during the steady state. In order to increase RHC of a DN, many methods have been presented. Ref. [2] presents an active distribution management (ADM) method to maximize PV hosting capacity by coordinated operation of control devices and PV inverters. Ref. [3] investigates PV penetration increment in low voltage (LV) network by utilizing on load tap charger (OLTC), reactive power compensation (RPC) and hybrid voltage control strategy (OLTC & RPC). Ref. [4] presents a control approach based on the voltage sensitivity analysis to prevent overvoltage occurrence and increase the PV hosting capacity of LV grids by determining dynamic set points for distributed electrical energy storage systems (EESS) management. In Ref. [5], RHC is improved by modifying the operation parameters of existing components, including on load tap changer (OLTC) and static var compensators (SVCs). Ref. [6] proposes a multi-period OPF-based method to evaluate the RDG hosting capacity, and this method incorporates the ANM schemes including coordinated voltage control, power factor control, and renewable energy curtailment. Based on these pervious works, we endeavor to enhance RHC in a more effective and efficient way.

One major obstacle that limits RHC is induced voltage rise because of the reverse power flow caused by renewable power integration [2]. In this regard, we consider coordinated operation of energy storage system (ESS) and static var compensator (SVC) to improve RHC. This is because ESS is able to decrease reverse active power and SVC is capable of voltage regulation by providing or releasing reactive power. Besides, ESS is widely used in electrical network and the operation cost of it is becoming reasonable and SVC can compensate or consume reactive power continuously with fast reaction in response to the voltage violation. Therefore, to improve RHC, we propose a method which incorporates optimal operation of ESS and SVC.

In this paper, we model PV hosting capacity and wind power hosting capacity as decision variables in an optimization fashion. The ESS schedule and SVC operation are also set as decision variables which are optimized with RHC. Specifically, we propose a day-ahead stochastic ESS scheduling model to maximize PV hosting capacity and wind power hosting capacity considering uncertainties of PV out, wind power output and load demand. Then we feed the results of the day-ahead stage into the real-time SVC operation model, which is to minimize the daily operation cost of DN while maintaining the reliable and secure operation of the system. In addition, we present an efficient solution approach based on Benders decomposition to solve the proposed day-ahead stochastic problem which is intractable due to time-coupling constraints. The effectiveness of the proposed model and the solution method is verified on the modified 33-node distribution system.

The remaining part of this paper is organized as follows. Section II provides the day-ahead stochastic ESS scheduling model. Section III introduces the real-time SVC operation model. Section IV presents the case studies to investigate the effectiveness of the proposed model and solution method. Section V includes concluding remarks.

II. Day-Ahead Stochastic ESS Scheduling Model

Day-ahead scheduling of ESS is essential for the economic and efficient operation of a DN. In this paper, the day-ahead stage is aimed at improving RHC via optimal scheduling of ESS. The schedule of the charging or discharging energy of ESS is decided in the day-ahead stage rather than real-time optimization. This is mainly due to two reasons, one is that the state of charge (SOC) of the ESS is time-coupled, and the other is that it is more feasible to obtain the optimal solution of ESS scheduling within the time horizon of 24 hours. Besides, without the day-ahead stage, the ESS charging or discharging decisions are made based on the information of the current time periods, which may lead to a poor performance on RHC enhancement. For example, the ESS can storage surplus renewable energy at certain time periods and the stored energy can be used at other time periods, which improves the operation flexibility and benefits the RHC enhancement. Thus, day-ahead optimization of ESS scheduling is more suitable for improving RHC which provides input data for the next real-time optimization stage.

A. Uncertainties Consideration

When dealing with day-ahead ESS scheduling, we consider three sources of uncertainty in the DN, e.g. PV energy output, wind energy output and load demand. In order to deal with these three uncertainties, we use stochastic programming to model the proposed ESS scheduling problem. To represent the uncertainties, numerous daily scenarios are sampled by using historical data from [7] and then reduced by a well-established backward-reduction algorithm proposed in [8] to a set of representative scenarios, which are fed into the day-ahead stage.

B. DN Model

In order to avoid the violations of voltage magnitude and line capacity when improving RHC, piecewise linearized DistFlow equations [9] is utilized since voltage magnitude and quadratic active/reactive power are considered as independent terms in these equations. The set of power flow, ESS charging/discharging, PV output, wind power output and reactive power facilities in a DN is characterized as,

$$\begin{aligned} P_{jtw} = & \sum_{i\in\varphi(j)} P_{itw} + \sum_{i\in\varphi(j)} r_{ij}\frac{P_{itw}^{qt}+Q_{itw}^{qt}}{V_0^2} + p_{jtw} - \sum_{m\in\Omega^{pv}(j)} P_m^{pv} \\ & - \sum_{n\in\Omega^{wt}(j)} P_n^{wt} - \sum_{l\in\Omega^{ess}(j)} P_{lt}^{ess}, \forall j,t,w \end{aligned} \tag{1a}$$

$$Q_{jtw} = \sum_{i\in\varphi(j)} Q_{itw} + \sum_{i\in\varphi(j)} x_{ij}\frac{P_{itw}^{qt}+Q_{itw}^{qt}}{V_0^2} + q_{jtw} + \sum_{b\in\Omega^{svc}(j)} Q_{btw}^{svc}, \quad \forall j,t,w \tag{1b}$$

$$V_{jtw} = V_{itw} + \frac{r_{ij}P_{itw} + x_{ij}Q_{itw}}{V_0}, \forall i\in\varphi(j), \forall j,t,w \tag{1c}$$

$$V_{jtw} = V^{sub}, j=1, \forall t,w \tag{1d}$$

where (1a) and (1b) describe the active and reactive power balance at each node, respectively. (1c) describes the voltage magnitude relationship between two adjacent nodes and (1d) gives the voltage of the substation. Note that the quadratic terms P_{jtw}^{qt} and Q_{jtw}^{qt} are employed to estimate P_{jtw}^2 and Q_{jtw}^2. P_{jtw}^{qt} and Q_{jtw}^{qt} can be estimated by applying piecewise linearization approximation (PLA) approach [10] with two auxiliary inequations, given as follows,

$$P_{jtw}^{qt} \geq M_{\lambda jtw}^{ap} P_{jtw} + N_{\lambda jtw}^{ap}, \forall \lambda \in \Omega^{ap}, \forall j, t, w \quad (2a)$$

$$Q_{jtw}^{qt} \geq M_{\lambda jtw}^{rp} Q_{jtw} + N_{\lambda jtw}^{rp}, \forall \lambda \in \Omega^{rp}, \forall j, t, w \quad (2b)$$

where $M_{\lambda jtw}^{AP}$, $M_{\lambda jtw}^{RP}$, $N_{\lambda jtw}^{AP}$ and $N_{\lambda jtw}^{RP}$ are the constant coefficients of the PLA approach, detailed information of these coefficients can be found in [9].

C. Scheduling Model of the ESS

The objective function of the day-ahead scheduling model is formulated as,

$$\begin{aligned} \text{Max} & \sum_{m \in \Omega^{pv}(j)} E_m^{pv} + \sum_{n \in \Omega^{wt}(j)} E_n^{wt} \\ & - \sum_{w \in W} \sum_{t \in T} \sum_{j \in J} c^{penalty} (s_{jtw}^{v,lb} + s_{jtw}^{v,ub} + s_{jtw}^{pq}) \end{aligned} \quad (3a)$$

$$\text{s.t.} \quad (1a)\text{-}(1d), (2a)\text{-}(2b) \quad (3b)$$

$$E_m^{pv} \geq 0, \forall m \quad (3c)$$

$$E_n^{wt} \geq 0, \forall n \quad (3d)$$

$$P_{mtw}^{pv} = \mu_{tw}^{pv} E_m^{pv}, \forall m, t, w \quad (3e)$$

$$P_{ntw}^{wt} = \mu_{tw}^{wt} E_n^{wt}, \forall n, t, w \quad (3f)$$

$$P_{lt}^{ess} = P_{lt-1}^{ess} + \eta^{ch} P_{lt}^{ch} - \eta^{dch} P_{lt}^{dch}, \forall l, t \quad (3g)$$

$$P^{ess,\min} \leq P_{lt}^{ess} \leq P^{ess,\max}, \forall l, t \quad (3h)$$

$$0 \leq P_{lt}^{ch} \leq \alpha_{lt}^{ch} P^{ch,\max}, \alpha_{lt}^{ch} \in \{0,1\}, \forall l, t \quad (3i)$$

$$0 \leq P_{lt}^{dch} \leq \alpha_{lt}^{dch} P^{dch,\max}, \alpha_{lt}^{dch} \in \{0,1\}, \forall l, t \quad (3j)$$

$$\alpha_{lt}^{ch} + \alpha_{lt}^{dch} = 1, \forall l, t \quad (3k)$$

$$Q^{svc,\min} \leq Q_{btw}^{svc} \leq Q^{svc,\max}, \forall b, t, w \quad (3l)$$

$$V_{jtw} + s_{jtw}^{v,lb} \geq V^{\min}, \forall j, t, w \quad (3m)$$

$$V_{jtw} + s_{jtw}^{v,ub} \leq V^{\max}, \forall j, t, w \quad (3n)$$

$$P_{jtw}^{qt} + Q_{jtw}^{qt} - s_{jtw}^{pq} \leq LC_{ij}^2, \forall j \in J \setminus 1, \forall t, w \quad (3o)$$

$$P_{jtw}^{qt} + Q_{jtw}^{qt} - s_{jtw}^{pq} \leq SC^2, j = 1, \forall t, w \quad (3p)$$

$$s_{jtw}^{v,lb}, s_{jtw}^{v,ub}, s_{jtw}^{pq} \in R^+, \forall j, t, w \quad (3q)$$

where the objective function (3a) consists of three terms, the first one is PV hosting capacity, the second one is wind power hosting capacity, and the third one is penalty cost for voltage violations and line capacity violations. Note that we introduce three non-negative slack variables $s_{jtw}^{v,lb}$, $s_{jtw}^{v,ub}$ and s_{jtw}^{pq} to denote overvoltage violation, undervoltage violation and over line capacity violation, respectively. Since $c^{penalty}$ is a very large constant which is imposed to avoid these violations, the third term should converge to zero without violations occurrence of voltage and line capacity.

(3b) describes the constraints of DN power flow and active/reactive quadratic term linearization. (3c) and (3d) guarantee that PV hosting capacity and wind power hosting capacity are non-negative. PV power output factor $\mu_{tw}^{pv} \in [0,1]$ and wind power output factor $\mu_{tw}^{wt} \in [0,1]$ appearing in (3e) and (3f) are utilized to capture the PV output and wind power output, respectively. (3g)-(3k) model the ESS charging and discharging process. Specifically, (3g) calculates the SOC of ESS for each time period considering efficiencies of charging and discharging. (3h) gives rated limits of the SOC of ESS. (3i) and (3j) describe the allowable charging/discharging power output of ESS. (3k) is added to avoid simultaneous charging and discharging. (3l) limits the reactive power compensation level of SVC devices. (3m) and (3n) give the relaxed voltage constraints. (3o) and (3p) show the relaxed DN line capacity constraints and the substation capacity constraints, respectively. (3q) guarantees that the introduced slack variables are non-negative.

D. Solution Methodology

The proposed day-ahead stochastic ESS scheduling problem is intractable because of time-coupling constraints, thus, it cannot be solved directly by using commercial solvers. In this regard, we develop Benders decomposition based on solution method to solve this problem. Specifically, the stochastic EES scheduling problem is decomposed into a master problem corresponding to RHC maximization problem and subproblems corresponding to penalty cost minimization problems. Note that each subproblem is associated with one scenario. Besides, Benders cuts are generated to link the master problem and subproblems.

1) Subproblem

In the scenario w with all variables affiliated with the iteration v, the subproblem is formulated as,

$$Z_w^{sub(v)} := \text{Min} \sum_{t \in T} \sum_{j \in J} c^{penalty} (s_{jtw}^{v,lb} + s_{jtw}^{v,ub} + s_{jtw}^{pq}) \quad (4a)$$

$$\text{s.t. } (3b), (3e)\text{-}(3q) \quad (4b)$$

$$E_m^{pv(v)} = E_m^{pv,\, fixed} : \lambda_{mtw}^{pv(v)}, \forall m \in \Omega^{pv}(j), \forall t, w \quad (4c)$$

$$E_n^{wt(v)} = E_n^{wt,\, fixed} : \lambda_{ntw}^{wt(v)}, \forall n \in \Omega^{wt}(j), \forall t, w \quad (4d)$$

Objective function (4a) is to minimize the penalty cost for violations of voltage as well as DN line capacity. (4c) and (4d) fix the variables $E_m^{pv(v)}$ and $E_n^{wt(v)}$ to the given values acquired from the master problem. The upper bound $Z^{upper(v)}$ for the optimal solution of the original problem (3) in iteration v is given as,

$$Z^{upper(v)} = \sum_{w \in W} \rho_w Z_w^{sub(v)} - \sum_{m \in \Omega^{pv}(j)} E_m^{pv} - \sum_{n \in \Omega^{wt}(j)} E_n^{wt} \quad (5)$$

Dual variables $\lambda_{mtw}^{pv(v)}$ and $\lambda_{ntw}^{wt(v)}$ of the variables $E_m^{pv(v)}$ and $E_n^{wt(v)}$ are adopted to obtain sensitivities to build Benders cuts. These sensitivities can be calculated by the following equations,

$$\lambda_m^{pv(v)} = \sum_{w \in W} \rho_w \sum_{t \in T} \lambda_{mtw}^{pv(v)}, \forall m \in \Omega^{pv}(j), \forall t, w \quad (6a)$$

$$\lambda_n^{wt(v)} = \sum_{w \in W} \rho_w \sum_{t \in T} \lambda_{ntw}^{wt(v)}, \forall n \in \Omega^{wt}(j), \forall t, w \quad (6b)$$

2) Mater Problem

The master problem in each iteration v is written as,

$$Z^{lower(v)} := \text{Min } \kappa^{(v)} - \sum_{m \in \Omega^{pv}(j)} E_m^{pv(v)} - \sum_{n \in \Omega^{wt}(j)} E_n^{wt(v)} \quad (7a)$$

$$\text{s.t.} \quad (3c)\text{-}(3d) \quad (7b)$$

$$\begin{aligned} \kappa^{(v)} \geq & \sum_{w \in W} Z_w^{sub(\sigma)} + \sum_{m \in \Omega^{pv}(j)} \lambda_m^{pv(\sigma)} (E_m^{pv(v)} - E_m^{pv(\sigma)}) \\ & + \sum_{n \in \Omega^{wt}(j)} \lambda_n^{wt(\sigma)} (E_n^{wt(v)} - E_n^{wt(\sigma)}) \quad \sigma = 1, 2, ..., v-1 \end{aligned} \quad (7c)$$

$$\kappa^{(v)} \geq \kappa^{down} \quad (7d)$$

$$\kappa^{(v)} - \sum_{m \in \Omega^{pv}(j)} E_m^{pv(v)} - \sum_{n \in \Omega^{wt}(j)} E_n^{wt(v)} \leq Z^{opt} \quad (7e)$$

In this master problem (7), $Z^{lower(v)}$ denotes the lower bound for the optimal solution of the original problem (3) in iteration v. (7c) describes the Benders cuts which link the master problem and subproblems. (7d) is imposed to accelerate the convergence. (7e) ensures that $Z^{lower(v)}$ is no larger than the minimum upper bound Z^{opt}.

3) Benders Decomposition Algorithm Procedure

The proposed Benders decomposition algorithm procedure is shown as Algorithm 1.

Algorithm 1 Benders Decomposition Algorithm

Step 1. Initialization: Set the iteration counter $v = 0$ and the minimum upper bound $Z^{opt} = \infty$. Solve the master problem (7) and obtain the initial values of $E_m^{pv(0)}$ and $E_n^{wt(0)}$. Then set $v = 1$, $Z^{upper(v)} = \infty$, $Z^{lower(v)} = -\infty$, $E_m^{pv,fixed} = E_m^{pv(0)}$ and $E_n^{wt,fixed} = E_n^{wt(0)}$. Set a small tolerance ε to control convergence.

Step 2. Iteration: Solve the subproblem (4) for each scenario w. Calculate the upper bound $Z^{upper(v)}$ according to (5).

Step 3. Minimum upper bound update: If $Z^{upper(v)} \leq Z^{opt}$, update the global solution $Z^{opt} = Z^{upper(v)}$.

Step 4. Convergence check: If $\left|Z^{upper(v)} - Z^{lower(v)}\right| \leq \varepsilon$, then terminate with the optimal solution. Otherwise, calculate the sensitivities by equations (11a) and (11b) to build the next Benders cut. Then, set $v \leftarrow v + 1$.

Step 5. Solve master problem: Solve the master problem (7), calculate $Z^{lower(v)}$ and update the values of $E_m^{pv(v)}$ and $E_n^{wt(v)}$. Then go back to the step 2 and continue.

III. Real-Time Optimal SVC Operation Model

The primary goal of real-time stage is to minimize daily operation cost of the DN via optimal SVC operation when achieving RHC enhancement. After obtaining results of ESS scheduling and RHC from the day-ahead stage, we feed these results into the real-time stage. The real-time optimization model is formulated as follows,

$$\begin{aligned} \text{Min} \ & \sum_{t\in\Omega(t)} \beta_t \sum_{i\in\varphi(j)} r_{ij} \frac{P_{it}^{qt} + Q_{it}^{qt}}{V_0^2} \\ & + \sum_{t\in\Omega(t)} \sum_{j\in\Omega(j)} c^{penalty}(s_{jt}^{v,lb} + s_{jt}^{v,ub} + s_{jt}^{pq}) \end{aligned} \tag{8a}$$

s.t. (1a)-(1d), (2a)-(2b), (3l)-(3q) (8b)

The objective function (8a) is to minimize the total daily DN operation cost, in which the first term represents the total daily active power loss cost and the second term represents the penalty cost for violations of voltage magnitude and DN line capacity. (8b) summarizes the DN power flow constraints, linearization of quadratic active/reactive power term, and DN operation constraints, respectively.

IV. Case Study

In this section, the IEEE 33-node distribution system is adopted to test the performance of the proposed methodology. Fig. 3 shows the test system. We assume PV generators are installed on candidate seven locations, namely nodes 5, 10, 16, 21, 23, 27 and 32, and wind energy generators are installed on four candidate locations, namely nodes 8, 14, 19 and 30. Details about the test system can be found in [11]. Per-unit value is used in case studies. The base values of power and voltage are set as 1 MVA and 12.66 kV, respectively. In the day-ahead stage, one thousand scenarios are sampled from the history data and nine representative ones are considered in the model. In the real-time stage, we assume that each PV system operates at the maximum power point so as to harvest as much PV energy as possible, and we also assume that load demand factor and wind energy output factor follows their average values, respectively. We employ CVX [12] and GUROBI [13] to solve the proposed problem.

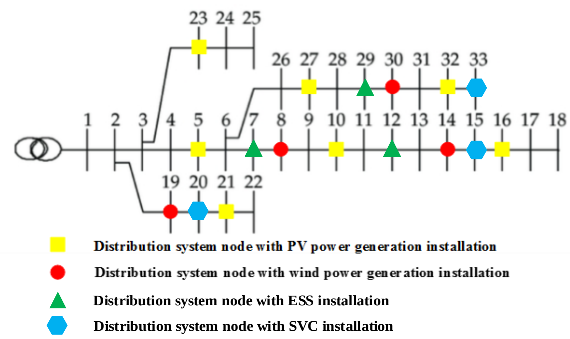


Fig. 1. Modified IEEE 33-node distribution system.

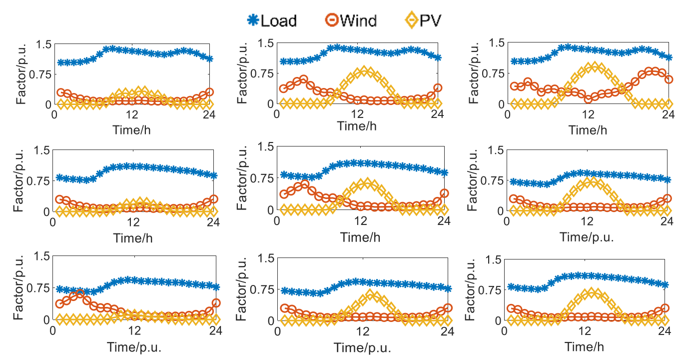


Fig. 2. Representative load-PV-wind scenarios considered in day-ahead stage.

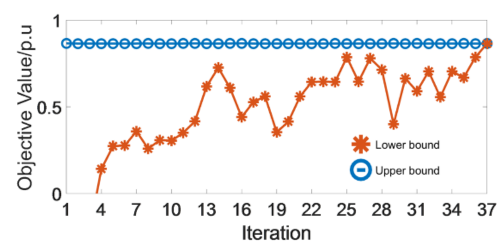


Fig. 3. Evolution of Benders decomposition based algorithm.

TABLE I

Computation Time of Solving the Proposed Day-Ahead Problem

Scenario Number	CVX-GUROBI	CVX_BD-GUROBI	
	[minutes]	[minutes]	[iterations]
9	NA	137	37

TABLE II

Results of RHC

PV energy hosting capacity(p.u.)	Wind energy hosting capacity(p.u.)
0.37	0.49

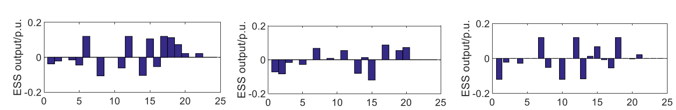
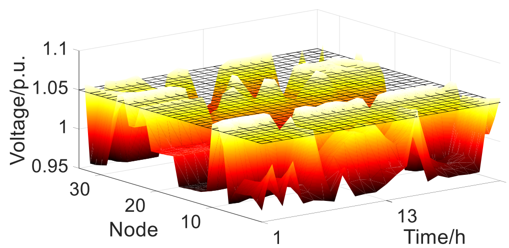


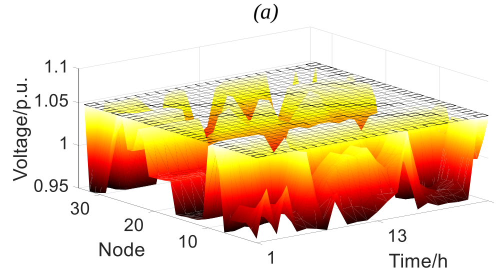
Fig.4. 24-hour charging/discharging schedule of each ESS in per unit.

Fig. 3 depicts the convergence performance of the proposed Benders decomposition based solution method for solving the day-ahead ESS scheduling model. Table I shows that the original problem (3) cannot be solved directly by using the solver GUROBI because of high computation complexity, but our proposed algorithm is capable of solving the same problem efficiently.

The optimal results of PV hosting capacity and wind power hosting capacity are given in Table II. Besides, Fig 4 shows the corresponding ESS charging/discharging schedule. As shown in Fig 4, the ESSs are being charged to store energy for later use when the net load is low, such as from hour 1 to hour 5. With the increasing net load, the ESSs are being discharged to help the renewable generators to meet the load demand, especially from hour 17 and hour 20.



(a)



(b)

Fig. 5 Comparison on voltage magnitude of (a) deterministic model and (b) our proposed model under the critical scenario.

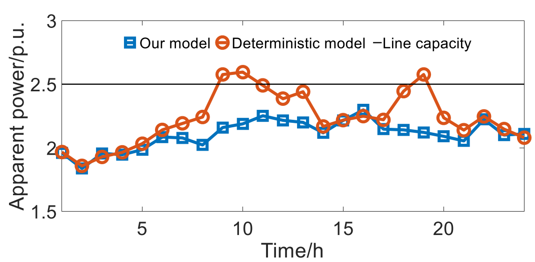


Fig. 6 Comparison on apparent power from node 10 to node 11 under the critical scenario.

The deterministic operation scheme is used as benchmark here. The formulation of the deterministic optimal SVC planning problem is similar to (3) and (8) but with only one scenario where the PV output, wind power output and load demand are substituted to their expected values. To compare the performance of our results and benchmark results, we employ a critical scenario with the high PV output factor and wind energy output factor but low load demand level. Fig 5 and Fig 6 compare the voltage profiles and apparent power under this critical scenario, respectively. It can be seen from these two figures that there are violations of voltage magnitude and line capacity with deterministic operation result while these violations cannot be observed with our result. This is because our model takes uncertainties into consideration in the day-ahead stage. Therefore, our model shows a better performance on voltage profile as well as apparent power under the critical scenario, and its results are more comprehensive and robust when addressing the uncertainties of the DN.

V. Conclusion

This paper presents a novel method to maximize RHC of a DN via optimal coordinated operation of ESS and SVC. In detail, we model PV hosting capacity and wind hosting capacity as decision variables in an optimization fashion. The ESS scheduling and SVC operation are also set as decision variables which are optimized with RHC. Specifically, we propose a day-ahead stochastic ESS scheduling model to maximize PV hosting capacity and wind hosting capacity considering uncertainties of PV out, wind power output and load demand. Then we feed the results of the day-ahead stage into the real-time SVC operation optimization model, which is to minimize the DN daily operation cost while maintaining the reliable and secure operation of the system. In addition, we present an efficient solution approach based on Benders decomposition to solve the proposed day-ahead stochastic problem which is intractable due to time-coupling constraints. The effectiveness of the proposed model and the solution method is verified on the modified 33-node distribution system.

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